unions of chains of associative relations

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The class ASSOCIATIVE of associative relations is closed under unions of chains.

a general theorem

Theorem.

Lemma.

Lemma.
Theorem.

Corollary. Eliminating the variable `t` from the theorem yields this corollary.

**application: Uchains[ASSOCIATIVE]**

Lemma.

```
In[12]:= SubstTest[Uchains, Uclosure[t],
    t -> fix[composite[inverse[IMAGE[x]], IMAGE[y]]]] // Reverse
```

```
Out[12]= Uchains[fix[composite[inverse[IMAGE[x]], IMAGE[y]]]] ==
    fix[composite[inverse[IMAGE[x]], IMAGE[y]]]
```

```
In[13]:= Uchains[fix[composite[inverse[IMAGE[x]], IMAGE[y]]]] :=
    fix[composite[inverse[IMAGE[x]], IMAGE[y]]]
```

Lemma.
Main Theorem. The union of a chain of associative relations is an associative relation.

In[18]:- (SubstTest[implies, and[equal[Uchains[u], u], equal[Uchains[v], v]],
   equal[Uchains[intersection[u, v]], intersection[u, v]],
   {u -> fix[image[inverse[CART], fix[composite[inverse[IMAGE[x]], IMAGE[y]]]]],
   v -> fix[image[inverse[CART], fix[composite[inverse[IMAGE[x]], IMAGE[y]]]]]} // Reverse) /
   {x -> composite[SWAP, RIF, cross[Id, composite[cross[SWAP, Id], inverse[RIF]]]],
   y -> composite[SWAP, RIF,
   cross[Id, composite[cross[inverse[ASSOC], SWAP], inverse[RIF]]]]}

Out[18]= equal[ASSOCIATIVE, Uchains[ASSOCIATIVE]] == True

In[19]:- Uchains[ASSOCIATIVE] := ASSOCIATIVE