CORE[image[S,singleton[x]]]

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In[1]:= << goedel57.04b; << tools.m

It is now: 2004 May 5 at 10:45
Loading Simplification Rules

weightlimit = 40

summary

A characterization of CORE functions is applied in this notebook to the derivation of a formula for CORE[image[S,singleton[x]]]. The result was originally discovered in a different, more roundabout way, but the derivation found earlier was quite messy and unenlightening. There is still some messiness in the present derivation, but the overall strategy is easier to understand.

definitions

In[2]:= idempotent[x_] := equal[composite[x, x], x]
In[3]:= total[x_] := equal[domain[x], V]

The strategy is to use this theorem which characterizes CORE functions:

In[4]:= implies[and[idempotent[x], total[x], subclass[x, inverse[S]],
   subcommute[x, S], FUNCTION[x], equal[x, CORE[fix[x]]]]]


two messy lemmas

In[5]:= union[cart[complement[image[S, singleton[x]]]], singleton[0]],
   id[image[S, singleton[x]]]] // idempotent // AssertTest
Out[5]= equal[union[cart[complement[image[S, singleton[x]]]], singleton[0]],
   id[image[S, singleton[x]]]], union[cart[complement[image[S, singleton[x]]]],
   union[cart[complement[image[S, singleton[x]]]], singleton[0]]],
   union[intersection[image[S, singleton[x]]], singleton[0]],
   intersection[cart[image[V, x], singleton[0]]],
   id[image[S, singleton[x]]]] = True

In[6]:= (%) /. x -> x_ / . Equal -> SetDelayed
results that can be made into permanent rules

In[9]:= FUNCTION[union[cart[complement[x], singleton[0]], id[x]]] // AssertTest
Out[9]= FUNCTION[union[cart[complement[x], singleton[0]], id[x]]] := True

In[10]:= FUNCTION[union[cart[complement[x_], singleton[0]], id[x_]]] := True

In[11]:= CORE[union[x, singleton[0]]] // RelnNormality
Out[11]= CORE[union[x, singleton[0]]] = CORE[x]

In[12]:= CORE[union[x_, singleton[0]]] := CORE[x]

In[13]:= SubstTest[implies, and[idempotent[w], total[w], subclass[w, inverse[S]],
subcommute[w, S], FUNCTION[w], equal[w, CORE[fix[w]]],
w -> union[cart[complement[ image[S, singleton[x]]], singleton[0]],
id[ image[S, singleton[x]]]]]
union[cart[complement[ image[S, singleton[x]]], singleton[0]],
id[ image[S, singleton[x]]]]] = True

In[14]:= CORE[ image[S, singleton[x_]]] := union[cart[complement[ image[S, singleton[x]]], singleton[0]], id[ image[S, singleton[x]]]]