an equality rule for PAIR[x, y]

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```
In[1]:= SetDirectory["l:"]; << goedel.08aug29a;<< tools.m

        :Package Title: goedel.08aug29a             2008 August 29 at 11:30 p.m.

        It is now:  2008 Aug 31 at 12:17

        Loading Simplification Rules

        TOOLS.M                                    Revised 2008 July 5

        weightlimit = 40
```

summary

The **GOEDEL** program has two ordered pairs, which agree for sets, but not for proper classes. The term **pair[x,y]** is a primitive undefined term whose properties are consistent with Kuratowski's ordered pair. The other ordered pair is a defined quantity:

```
In[2]:= A[cart[set[x], set[y]]]

Out[2]= PAIR[x, y]
```

In this notebook an equality rule for **PAIR[x, y]** is derived. A counterexample is presented to show that this equality rule differs from the equality rule for **pair[x, y]**.

derivation

Lemma.

```
In[3]:= SubstTest[implies, equal[w, z],
          equal[cart[set[first[w]], set[second[w]]], cart[set[first[z]], set[second[z]]]],
          {w → PAIR[u, v], z → PAIR[x, y]}] // Reverse

Out[3]= or[equal[cart[set[u], set[v]], cart[set[x], set[y]]],
          not[equal[PAIR[u, v], PAIR[x, y]]] == True

In[4]:= (% /. {u → u_, v → v_, x → x_, y → y_}) /. Equal → SetDelayed
```

Lemma.

```
In[5]:= SubstTest[implies, equal[w, z], equal[A[w], A[z]],
          {w -> cart[set[u], set[v]], z -> cart[set[x], set[y]]}] // Reverse

Out[5]= or[equal[PAIR[u, v], PAIR[x, y]],
          not[equal[cart[set[u], set[v]], cart[set[x], set[y]]]] == True

In[6]:= (% /. {u → u_, v → v_, x → x_, y → y_}) /. Equal → SetDelayed
```

Theorem.

```
In[7]:= equiv[equal[cart[set[u], set[v]], cart[set[x], set[y]]], equal[PAIR[u, v], PAIR[x, y]]]

Out[7]= True

In[8]:= equal[PAIR[u_, v_], PAIR[x_, y_]] := equal[cart[set[u], set[v]], cart[set[x], set[y]]]
```

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a counterexample

The equality rule for **PAIR** differs from that for **pair**. The latter is:

```
In[9]:= equal[pair[u, v], pair[x, y]]

Out[9]= and[equal[set[u], set[x]], equal[set[v], set[y]]]
```

We do have an implication in one direction:

```
In[10]:= implies[equal[pair[u, v], pair[x, y]], equal[PAIR[u, v], PAIR[x, y]]]

Out[10]= True
```

In the other direction, the following provides a counterexample.

```
In[11]:= implies[equal[PAIR[u, v], PAIR[x, y]], equal[pair[u, v], pair[x, y]]] /.
           {u → V, v → V, x → 0, y → V}

Out[11]= False
```