a counterexample related to FULL

Johan G. F. Belinfante
2006 January 8

In [1]:= SetDirectory["l: "]; \<< goedel77.07a; \<< tools.m

:Package Title: goedel.07a 2006 January 7 at 1:40 p.m.
It is now: 2006 Jan 8 at 4:51
Loading Simplification Rules
TOOLS.M Revised 2006 January 2
weightlimit = 40

summary

A counterexample is used to show that the class of full sets does not contain its own power class.

the ensemble tool

The ensemble tool in the file tools.m is useful for generating finite counterexamples.

In[3]:= Begin["Goedel\`Private\""];

In[5]:= ?? bits

bits[n] is a list of the bits of n in reversed order
bits[0] := {};
bits[n_] := Join[{Mod[n, 2]}, bits[Quotient[n, 2]]]

In[4]:= ?? ens

ens[n] is the n-th set in the cumulative hierarchy (for finite n)
ens[0] := 0
ens[n_] := union@@(set[ens[#1-1]] 4) \[\Cap\] Flatten[Position[bits[n], 1]]
counterexample

Cantor's theorem implies that $P[x]$ cannot be a subclass of $x$ when $x$ is a set. This does not generalize to proper classes. In particular, the universal class $V$, the Russell class $RUSSELL$ and the class $REGULAR$ of all regular sets all have this property.

```
Out[6] = {True, True, True}
```

The class $FULL$ of all full sets is a proper class.

```
In[8] := full[x]
Out[8] = subclass[U[x], x]
In[9] := class[x, full[x]]
Out[9] = FULL
In[10] := member[FULL, V]
Out[10] = False
```

In this section a counterexample is used to show that $P[FULL]$ is not a subclass of $FULL$. This counterexample could be discovered using the `ensemble` tool as follows:

```
In[7] := Select[Map[ens, Range[5]], member[#, dif[P[FULL], FULL]] &]
Out[7] = {set[set[0]]}
```

With this counterexample in hand, one readily establishes that the power class of $FULL$ is not a subclass of $FULL$.

```
In[11] := SubstTest[and, member[x, y], empty[y], {x -> set[set[0]], y -> dif[P[FULL], FULL]}]
```