The function RCF

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\[
\text{\texttt{\textless \textless goedel52.m82; \textless \textless tests.m}}
\]

:Package Title: GOEDEL52.M82 2002 February 4 at 1:30 a.m.

It is now: 2002 Feb 4 at 15:47

Loading Simplification Rules

TESTS.M Revised 2002 February 3

weightlimit = 40

Context switch to 'Goedel'Private is needed for ReplaceTest

Just ignore the error message about Unterminated use of BeginPackage

\texttt{Get::bebal: Unterminated uses of BeginPackage or Begin in \textless \textless tests.m.}

\section{Introduction}

A new function \texttt{RCF} has been added to the \texttt{GOEDEL} program. The membership rule used to define this function is:

\[
\text{member}[x, \text{RCF}]
\]

\[
\text{and}[\text{equal}[	ext{RC[first]}[x]], \text{second}[x]], \text{member}[\text{first}[x], V]]
\]

All other properties of this function have been deduced from this membership rule, using the \texttt{GOEDEL} program itself. In this notebook we summarize a few of these properties.

\section{Review: the function \texttt{RC[x]}.}

The function \texttt{RC[x]} introduced earlier performs relative complements with respect to \texttt{x}:

\[
\text{FUNCTION[RC[x]]}
\]

\[
\text{True}
\]

The domain of \texttt{RC[x]} is the power set \texttt{P[x]} provided that \texttt{x} is a set.

\[
\text{domain[RC[x]]}
\]

\[
\text{intersection[image[V, singleton[x]], P[x]]}
\]
A nice characterization of \( RC[x] \) is:

\[
\text{class}[\text{pair}[u, v], \text{and}[	ext{disjoint}[u, v], \text{equal}[\text{union}[u, v], x]]] = RC[x]
\]

This function is its own inverse:

\[
\text{inverse}[RC[x]] = RC[x]
\]

For any subset of \( x \), a second application of relative complementation takes one back.

\[
\text{composite}[RC[x], RC[x]] = \text{id}[\text{intersection}[\text{image}[V, \text{singleton}[x]], P[x]]]
\]

Since the complement of a set is a proper class, the function \( RC[x] \) is the empty set when \( x \) is not a set.

\[
RC[V]
\]

\[
0
\]

Many formulas for \( RC[x] \) involve the class \( \text{image}[V, \text{singleton}[x]] \) which is either \( V \) or \( 0 \) depending on whether \( x \) is a set.

\[
\text{equal}[V, \text{image}[V, \text{singleton}[x]]] = \text{member}[x, V]
\]

\[
\text{equal}[0, \text{image}[V, \text{singleton}[x]]] = \text{not}[\text{member}[x, V]]
\]

## Characterizations of RCF

The function \( RCF \) takes \( x \) to \( RC[x] \).

\[
\text{lambda}[x, RC[x]] = RCF
\]

For any class \( x \), the function \( RC[x] \) is a set.

\[
\text{member}[RC[x], V] = True
\]

Consequently, the domain of \( RCF \) is the class \( V \) of all sets:
Each $\text{RC}[x]$ function is a bijection:

\[
\text{member}[	ext{RC}[x], \text{BIJ}]
\]

True

The range of $\text{RCF}$ is the class of all $\text{RC}[x]$ functions, and therefore:

\[
\text{subclass}[\text{range}[\text{RCF}], \text{BIJ}]
\]

True

A point $\text{pair}[u, v]$ belongs to some $\text{RC}[x]$ function provided $u$ and $v$ are disjoint.

\[
\text{U}[\text{range}[\text{RCF}]]
\]

$\text{DISJOINT}$

The relation $\text{DISJOINT}$ ia:

\[
\text{class}[\text{pair}[x, y], \text{equal}[0, \text{intersection}[x, y]]]
\]

$\text{DISJOINT}$

**Some useful observations.**

Many of the rules for $\text{RCF}$ were obtained from this description of it:

\[
\text{composite}[\text{IMAGE}[\text{id}[\text{DISJOINT}]], \text{VERTSECT}[\text{inverse}[\text{CUP}]]]
\]

$\text{RCF}$

Here $\text{CUP}$ is the function

\[
\text{lambda}[\text{pair}[x, y], \text{union}[x, y]]
\]

$\text{CUP}$

The functions $\text{VERTSECT}$ and $\text{IMAGE}$ are in general defined by

\[
\text{lambda}[y, \text{image}[x, \text{singleton}[y]]]
\]

$\text{VERTSECT}[x]$

\[
\text{lambda}[y, \text{image}[x, y]]
\]

$\text{IMAGE}[x]$

In particular, the function $\text{IMAGE}[\text{id}[x]]$ is
We call a relation thin if all its vertical sections are sets:

\[
\text{thin}[x] = \text{equal}[V, \text{domain}[\text{VERTSECT}[x]]]
\]

A key fact is that the inverse of \text{CUP} is a thin relation:

\[
\text{thin}[^{-1}\text{CUP}] = \text{True}
\]

The following indirect description avoids both \text{VERTSECT} and \text{IMAGE}, but gives only the composite with the inverse of the membership relation \text{E}.

\[
\text{composite}[\text{id}[^{-1}\text{DISJOINT}], \text{inverse}[^{-1}\text{CUP}]] = \text{composite}[\text{inverse}[\text{E}], \text{RCF}]
\]

\[
\text{True}
\]

The membership relation \text{E} is characterized by:

\[
\text{member}[\text{pair}[x, y], \text{E}]
\]

\[
\text{and}[\text{member}[x, y], \text{member}[y, V]]
\]