Theorem SP–DO–C

Johan G. F. Belinfante
2004 February 12

In[1]:= << godel54.11a; << tools.m

:Package Title: godel54.11a 2004 February 11 at 5:30 p.m.

It is now: 2004 Feb 13 at 15:48

Loading Simplification Rules

TOOLS.M Revised 2004 January 3

weightlimit = 40

summary

This notebook contains a two-step derivation of Theorem SP–DO–C, which was proved using Otter 1997 December 15 using essentially the same technique. The theorem says that if \( x \) is a set, then the domain of the complement of \( x \) is the universal class \( V \).

derivation

This first step can be carried out either by a blind application of AssertTest, or as follows:

In[2]:= SubstTest[equal, 0, dif[y, x], y -> cart[complement[domain[complement[x]]], V]] // Reverse


This made into a temporary rewrite rule.

In[3]:= (% /. x -> x_) /. Equal -> SetDelayed

The second step uses the subset theorem: a subclass of a set is a set.

In[4]:= Map[implies, member[x, y], #] &,
   SubstTest[implies, and[subclass[u, v], member[v, V]], member[u, V],
   {u -> cart[complement[domain[complement[x]]], V], v -> x}]

Out[4]= or[equal[V, domain[complement[x]]], not[member[x, y]]] = True

Because the following rewrite rule will be made permanent, an extra variable \( y \) has been introduced to facilitate pattern matching for applications of this rule.

In[5]:= or[equal[V, domain[complement[x_]]], not[member[x_, y_]]] := True