QUIZ 3 FOR MATH 2401 BUNIMOVICH

NO CALCULATORS, NOTES, OR BOOKS ALLOWED. SHOW ALL YOUR WORK.

Name: Student  
TA:  

1. Is the set \( \{ x \mid 0 \leq x < 1 \} \) open, closed, or neither? (5 pts.)

\textbf{Neither.}  
For the set to be closed, it must contain all of its boundary points (in this case, 0 and 1). To be open, it must contain none of its boundary points. Since this set contains 0 but not 1, it is neither open nor closed.

2. Does the function \( f(x,y) = \frac{x^2}{x^2+y^2} \) have a limit at the origin (0,0)? (10 pts.)

\textbf{No.}

\[
\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to0} f(0,y) = \lim_{y\to0} \frac{0}{y^2} = 0.
\]

Along \( x=0 \):

\[
\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to0} f(x,0) = \lim_{x\to0} \frac{x^2}{x^2} = 1.
\]

Along \( y=0 \):

Since these two paths produce different limits, no limit exists.