QUIZ 4 FOR MATH 2401 BUNIMOVICH

NO CALCULATORS, NOTES, OR BOOKS ALLOWED. SHOW ALL YOUR WORK.

Name: Solution  
TA: 

1) The intensity of light in a neighborhood of the point \((-1, 2)\) is given by a function of the form 
\[ f(x, y) = A - x^2 - 2y^2. \] 
Find the path followed by a light seeking particle that originates at the center of the neighborhood. (15 pts.)

The direction in which light intensity must rapidly is given by 
\[ \nabla f = (-2x, -4y). \]
Thus, the particle's direction will match this. If the particle's path is given by 
\[ \vec{r}(t) = (x(t), y(t)) \]
then its direction is given by 
\[ \vec{r}'(t) = (x'(t), y'(t)). \]

So we solve:

\[ \begin{align*} 
(i): & \quad x'(t) = -2x \\
(ii): & \quad y'(t) = -4y 
\end{align*} \]

\( (i) \) : 
\[ \frac{dx}{dt} = -2x \quad \Rightarrow \quad \frac{dx}{x} = -2dt \quad \Rightarrow \quad \ln x = -2t + C \]
\[ \Rightarrow \quad e^{-2t} C = x \]
\[ \Rightarrow \quad x = e^C e^{-2t} = x_0 e^{-2t} \]

So 
\[ x(t) = -e^{-2t} \]

Similarly, 
\[ y(t) = 2e^{-4t} \]

So the particle's path is given by:
\[ \vec{r}(t) = (-e^{-2t}, 2e^{-4t}). \]