1. Let $A$ be the region inside the rectangle with vertices $(0, 0), (2, 0), (2, 1),$ and $(0, 1)$. Let $B$ be the region inside the trapezoid with vertices $(0, 0), (3, 0), (2, 1),$ and $(0, 1).$ Let $C$ be the region inside the triangle with vertices $(1, 0), (3, 0),$ and $(2, 1).$ Finally, let $D$ be the region inside the triangle with vertices $(1, 0), (2, 0),$ and $(2, 1).$ We know that

$$\iint_A f(x,y) dA = 3,$$

$$\iint_B f(x,y) dA = 2,$$ and

$$\iint_C f(x,y) dA = -5.$$

Find the integral $\iint_D f(x,y) dA$, or explain carefully why there is insufficient information given.

Let’s draw a picture of all this.

Here $E$ is the triangle with vertices $(2, 0), (3, 0),$ and $(2, 1).$

First, note that

$$\iint_B f(x,y) dA = \iint_A f(x,y) dA + \iint_E f(x,y) dA$$

Thus,

$$\iint_E f(x,y) dA = \iint_B f(x,y) dA - \iint_A f(x,y) dA = 2 - 3 = -1.$$
Next, note that

\[ \iiint_{C} f(x,y) \, dA = \iint_{D} f(x,y) \, dA + \iint_{E} f(x,y) \, dA, \text{ or} \]

\[ \iiint_{D} f(x,y) \, dA = \iint_{C} f(x,y) \, dA - \iint_{E} f(x,y) \, dA \]

\[ = -5 - (-1) = -4. \]

2. Find the integral

\[ \int_{0}^{1} \int_{0}^{1} e^{y^3} \, dy \, dx. \]

I cannot think of an antiderivative for \( e^{y^3} \), so let’s find the two-dimensional integral that lead to this iterated integral and see if changing the order of integration helps.

A picture:

Now,

\[ \int_{0}^{1} \int_{0}^{1} e^{y^3} \, dy \, dx = \int_{0}^{1} e^{y^3} \, dx \, dy \]

\[ = \left. \int_{0}^{1} y^2 e^{y^3} \, dy \right|_{0}^{1} = \frac{1}{3} e - 1 \]

\[ = \frac{1}{3} (e - 1) \]

\textbf{Finis}