I. Homework to be handed in before 10:06 a.m., Wednesday, January 19

Let $S$ be the set of all vectors $(x, y, z)$ in $\mathbb{R}^3$ such that $x + y + z = 0$.

a) Show that $S$ is a subspace of $\mathbb{R}^3$.

b) Find a basis for $S$.

II. Homework to be handed in before 10:06 a.m., Wednesday, January 26

Explain carefully what is wrong with Theorem 3.10 (page 107 of the Apostol textbook).

III. Homework to be handed in before 10:06 a.m., Wednesday, February 3

In the real linear space $C(0, \pi)$ with inner product $(x, y) = \int_0^\pi x(t)y(t)dt$, let

$$x_n(t) = \sqrt{\frac{2}{\pi}} \sin nt.$$ 

a) Prove that $S_N = \{x_1, x_2, \ldots, x_N\}$ is an orthonormal set.

b) Find the element $y_N$ of the span of $S_N$ nearest to $f(t) = 1$.

c) Find $\|y_N - f\|^2$.

IV. Homework to be handed in before 10:06 a.m., Wednesday, February 9

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4, 3x_1 + 2x_2 + 2x_4, 7x_1 + 4x_2 - 2x_3 + 4x_4, 2x_1 + x_2 - x_3 + x_4).$$

a) Find a basis for the null space of $T$.

b) Find a basis for the range of $T$.
V. Homework to be handed in before 10:06 a.m., Wednesday, February 16

For each \( x \in \mathbb{R}^2 \), let \( T_\alpha(x) \) be the vector resulting from rotating \( x \) through an angle \( \alpha \) in a counterclockwise direction about the origin.

a) Find the matrix representation with respect to the usual basis \( \{i, j\} \) of the linear function \( T_\alpha : \mathbb{R}^2 \to \mathbb{R}^2 \) so defined.

b) Find the vector that results from rotating \( x = (14, -23) \) 60 degrees about the origin (counterclockwise).

c) Find the matrix representation of the composition \( T_\beta \circ T_\alpha \).

d) Observe that \( T_\beta \circ T_\alpha = T_{\alpha + \beta} \), and deduce a familiar trigonometric identity.

VI. Homework to be handed in before 10:06 a.m., Wednesday, February 23

Let

\[
A = \begin{bmatrix}
1 & 2 & 3 & 5 \\
-1 & 2 & 3 & 4 \\
2 & 2 & 5 & 6 \\
1 & 3 & 4 & 5
\end{bmatrix}.
\]

a) Find the determinant \( d(A) \) and prove you have the correct answer. (Telling me what Matlab or Maple or Mathematica, etc., says is not a proof.

b) If \( A \) has an inverse, find its determinant. Otherwise, explain carefully how you know \( A \) is not invertible. (Here also do not simply take the word of some computer program.)
VII. Homework to be handed in before 10:06 a.m., Wednesday, March 29

Let

\[ A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}. \]

a) Find a diagonal matrix similar to \( A \).

b) Use the result of part a) to find a square root of \( A \). In other words, find a matrix \( R \) so that \( R^2 = A \).

VIII. Homework to be handed in before 10:06 a.m., Wednesday, April 5

1. Exercise #7, page 159
2. Exercise #13, page 226

IX. Homework to be handed in before 10:06 a.m., Wednesday, April 19

Let \( S \) be the surface with equation \( x^2 + 2yz = 0 \), where \((x, y, z)\) are coordinates with respect to the standard basis for \( \mathbb{R}^3 \). Find an orthonormal basis \( E = \{u_1, u_2, u_3\} \) for \( \mathbb{R}^3 \) so that the coordinates \((\tilde{x}, \tilde{y}, \tilde{z})\) of the points of \( S \) with respect to \( E \) satisfy an equation in which there are no “cross product” terms—that is, the equation involves only \( \tilde{x}^2, \tilde{y}^2, \) and \( \tilde{z}^2 \). Identify and sketch the surface.
X. Homework to be handed in before 10:06 a.m., Wednesday, April 26

1. Suppose the square matrix \( A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \) has the property that

\[ |a_{ii}| > \sum_{\substack{j=1 \atop j \neq i}}^{n} |a_{ij}| \text{ for each } i = 1, 2, \ldots, n. \]

Explain how you know \( A \) is invertible.

2. Suppose \( A \) and \( B \) are \( n \times n \) matrices, and suppose \( A \) is invertible and \( |\lambda| < 1 \) for every eigenvalue \( \lambda \) of \( A^{-1}B \). Explain how you know the matrix \( A + B \) is invertible.