1. Suppose the square matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ has the property that 

$$|a_{ii}| \geq \sum_{j=1, j\neq i}^{n} |a_{ij}|$$

for each $i = 1, 2, \ldots, n$.

Explain how you know $A$ is invertible.

2. a) Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$ 

Find the condition number $\text{cond}(A)_\infty$ that is generated by the norm $| \cdot |_\infty$.

b) Give an example of a matrix $A$ such that $\text{cond}(A)_\infty < 1$, or explain carefully why there is no such matrix.

3. Suppose $A$ and $B$ are $n \times n$ matrices, and suppose $A$ is invertible and $\|A^{-1}B\| < 1$.

Explain how you know the matrix $A + B$ is invertible.

4. Find as small an upper bound as you can for the magnitude of the error incurred using linear interpolation in a table of $\tan x$ for $0 \leq x \leq 45^\circ$ in which there are equally spaced entries one degree apart.