Math 4320A                                    Final Examination                               June 13, 1997

You may use any books, notes, or calculators you wish. Write your answers so that someone other than yourself can understand your exposition. Fortuna vobiscum.

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1. Find all points at which the function has a derivative and find all points at which it is analytic. Explain.
   
a) \( f(z) = 2x(1 - y) + i(x^2 - y^2 + 2y) \).

b) \( f(z) = e^y e^{-ix} \).

2. Let \( f(z) = z^j \), where \( z^j \) denotes the principal branch of \( z^j \). Find
   
   \[ \int_L f(z) \, dz, \]
   
   where \( L \) is the line segment from \((1, 0)\) to \((2, 0)\). [Give your answer in rectangular form: \( a + ib \), where \( a \) and \( b \) are real.]

3. Let \( f(z) = \frac{1}{z^3(z - i)} \).
   
a) Find a Laurent series expansion in powers of \( z \) for \( f \), and specify the region for which it is valid.
   
b) Find another Laurent series expansion in powers of \( z \) for \( f \), and specify the region for which it is valid.

4. Let \( C \) be the circle \( C = \{ z : |z| = 1/2 \} \) oriented positively. Find
   
   \[ \int_C \frac{1}{z^4(z - i)} \, dz. \]

5. Find the Taylor series representation of \( f(z) = \frac{1}{1 - z} \) in powers of \( z + i \). For what values of \( z \) is this representation valid? Explain.

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