Computer Project Two

This project is designed to familiarize you with Newton's method. The main issues are:

(1)How fast does it converge?

(2)How close to the desired solution do you have to start in the first place?

To do this project, consider the following function:

$$f(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$$

where each of the constants A, B, C, D, E and F is plus or minus one. To decide which, toss a coin six times. If you get heads on the first toss, $A = 1$. I f you get tails on the first toss, $A = -1$. In the same way, determine the rest of the coefficients.

Write down the resulting polynomial $f(x)$, and use it in the rest of this project.

Next, graph the function using the applet that does Newton's method, listed below, and zoom out until you've determined how many roots there are.

By looking at the graph, determine how many roots there are, and give their approximate values to, say, one decimal place of accuracy.

Next go to the applet that finds which roots different starting points converge to for Newton's method, also listed below, and use is to answer the following:

For each of the roots x_j, find the values a_j and b_j so that for any x in the interval (a_j, b_j), Newton's method starting from x converges to x_j. Then for each j, compute $b_j - a_j$, the width of this interval of convergence. Finally compute $f''(x_j)$ at each of the

roots. Do you notice any relation between the numbers $f''(x_j)$ and the numbers $b_j - a_j$?

The applet that shows which roots different starting points converge to for Newton's method also shows how long it takes for the first six decimal places to converge. This is indicated by the depth of the colored lines though the point.

Find the points x_k with the slowest convergence in the interval $[-8, 8]$. What do you notice about $f'(x_k)$? Can you explain this? Test your hypothesis by going back to the applet that produces the sequence of Newton approximations, and see what that sequence is up to the point of convergence (for the 16 decimal places the computer keeps track of) when you start from one of these points which takes the most steps. What does the sequence of approximations do?