Supplemental problems: §2.2, §2.3

1. Put an augmented matrix into reduced row echelon form to solve the system
\[ x_1 - 2x_2 - 9x_3 + x_4 = 3 \]
\[ 4x_2 + 8x_3 - 24x_4 = 4. \]

2. We can use linear algebra to find a polynomial that fits given data, in the same way
that we found a circle through three specified points in the §2.1 Webwork.
Is there a degree-three polynomial \( P(x) \) whose graph passes through the points
\((-2, 6), (-1, 4), (1, 6), \) and \((2, 22)\)? If so, how many degree-three polynomials have
a graph through those four points? We answer this question in steps below.

a) If \( P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \) is a degree-three polynomial passing through
the four points listed above, then \( P(-2) = 6, \ P(-1) = 4, \ P(1) = 6, \) and
\( P(2) = 22. \) Write a system of four equations which we would solve to find \( a_0, \)
\( a_1, a_2, \) and \( a_3. \)

b) Write the augmented matrix to represent this system and put it into reduced
row-echelon form. Is the system consistent? How many solutions does it have?