Math 1553 Supplement §4.5, 5.1-5.3

1. **a)** Fill in: A and B are invertible $n \times n$ matrices, then the inverse of AB is ________.

b) If the columns of an $n \times n$ matrix Z are linearly independent, is Z necessarily invertible? Justify your answer.

c) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, does $Ax = 0$ necessarily have a unique solution? Justify your answer.

2. Let A be an $n \times n$ matrix.

a) Using cofactor expansion, explain why $\det(A) = 0$ if A has a row or a column of zeros.

b) Using cofactor expansion, explain why $\det(A) = 0$ if A has adjacent identical columns.

3. Find the volume of the parallelepiped in $\mathbf{R}^4$ naturally determined by the vectors

$$\begin{pmatrix} 4 \\ 1 \\ 3 \\ 8 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 7 \\ 0 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ -5 \\ 0 \\ 7 \end{pmatrix}.$$

4. If A is a 3×3 matrix and $\det(A) = 1$, what is $\det(-2A)$?

5. **a)** Is there a real 2×2 matrix A that satisfies $A^4 = -I_2$? Either write such an A, or show that no such A exists.
(hint: think geometrically! The matrix $-I_2$ represents rotation by π radians).

b) Is there a real 3×3 matrix A that satisfies $A^4 = -I_3$? Either write such an A, or show that no such A exists.