1.  a) Fill in: $A$ and $B$ are invertible $n \times n$ matrices, then the inverse of $AB$ is ________.

   b) If the columns of an $n \times n$ matrix $Z$ are linearly independent, is $Z$ necessarily invertible? Justify your answer.

   c) If $A$ and $B$ are $n \times n$ matrices and $ABx = 0$ has a unique solution, does $Ax = 0$ necessarily have a unique solution? Justify your answer.

2.  Let $A$ be an $n \times n$ matrix.

   a) Using cofactor expansion, explain why $\det(A) = 0$ if $A$ has a row or a column of zeros.

   b) Using cofactor expansion, explain why $\det(A) = 0$ if $A$ has adjacent identical columns.

3.  Find the volume of the parallelepiped in $\mathbb{R}^4$ naturally determined by the vectors

   $$\begin{pmatrix} 4 \\ 1 \\ 3 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ -5 \\ 0 \\ 7 \end{pmatrix}. $$

4.  If $A$ is a $3 \times 3$ matrix and $\det(A) = 1$, what is $\det(-2A)$?

5.  a) Is there a real $2 \times 2$ matrix $A$ that satisfies $A^4 = -I_2$? Either write such an $A$, or show that no such $A$ exists.  
   (hint: think geometrically! The matrix $-I_2$ represents rotation by $\pi$ radians).

   b) Is there a real $3 \times 3$ matrix $A$ that satisfies $A^4 = -I_3$? Either write such an $A$, or show that no such $A$ exists.