Math 1553 Supplement §6.1, 6.2

Supplemental Problems

1. Match the statements (i)-(v) with the corresponding statements (a)-(e). All matrices are 3×3. There is a unique correspondence. Justify the correspondences in words.

(i) $Ax = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ has a unique solution.
(ii) The transformation $T(v) = Av$ fixes a nonzero vector.
(iii) A is obtained from B by subtracting the third row of B from the first row of B.
(iv) The columns of A and B are the same; except that the first, second and third columns of A are respectively the first, third, and second columns of B.
(v) The columns of A, when added, give the zero vector.

(a) 0 is an eigenvalue of A.
(b) A is invertible.
(c) $\det(A) = \det(B)$
(d) $\det(A) = -\det(B)$
(e) 1 is an eigenvalue of A.

2. Find a basis $\mathcal{B}$ for the (-1)-eigenspace of $Z = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

3. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.

4. Give an example of matrices A and B which satisfy the following:
(I) A and B have the same eigenvalues, and the same algebraic multiplicities for each eigenvalue.
(II) For some eigenvalue λ, the λ-eigenspace for A has a different dimension than the λ-eigenspace for B.

Justify your answer.

5. Let $A = \begin{pmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{pmatrix}$. Find the eigenvalues of A.

6. Using facts about determinants, justify the following fact: if A is an $n \times n$ matrix, then A and A^T have the same characteristic polynomial.