MATH 1553, FALL 2018
SAMPLE MIDTERM 1: THROUGH SECTION 3.4

Please read all instructions carefully before beginning.

• You have 50 minutes to complete this exam.
• There are no aids of any kind (calculators, notes, text, etc.) allowed.
• Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
• You may cite any theorem proved in class or in the sections we covered in the text.
• Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is not meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.
Problem 1. [Parts a) through f) are worth 2 points each]

a) Compute: \[
\begin{pmatrix}
3 & 2 \\
-2 & 0 \\
1 & 4 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
-3 \\
\end{pmatrix} =
\]

The remaining problems are True or false. Circle T if the statement is always true, and circle F otherwise. You do not need to justify your answer.

b) T F The matrix \[
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
\end{pmatrix}
\] is in reduced row echelon form.

c) T F If \(Ax = b\) is consistent, then the equation \(Ax = 5b\) is consistent.

d) T F If the augmented matrix corresponding to a linear system of equations has a pivot in every row, then the system is consistent.

e) T F If \(A\) is an \(m \times n\) matrix and \(Ax = 0\) has a unique solution, then \(Ax = b\) is consistent for every \(b\) in \(\mathbb{R}^m\).

f) T F The three vectors \[
\begin{pmatrix}
1 \\
0 \\
0 \\
\end{pmatrix}, \begin{pmatrix}
1 \\
1 \\
0 \\
\end{pmatrix}, \begin{pmatrix}
-1 \\
0 \\
1 \\
\end{pmatrix}
\] span \(\mathbb{R}^3\).
Problem 2.

Parts (a) and (b) are 2 points each. Parts (c) and (d) are 3 points each.

a) If $A$ is a $2 \times 3$ matrix with 2 pivots, then the set of solutions to $Ax = 0$ is a:

(circle one answer) point line plane 3-plane

in:

(circle one answer) $\mathbb{R}$ $\mathbb{R}^2$ $\mathbb{R}^3$.

b) Write a vector equation which represents an inconsistent system of two linear equations in $x_1$ and $x_2$.

c) For some $2 \times 2$ matrix $A$ and vector $b$ in $\mathbb{R}^2$, the solution set of $Ax = b$ is drawn below. Draw the solution set of $Ax = 0$.

\begin{center}
\begin{tikzpicture}
\draw[help lines] (-4,0) grid (4,4);
\draw[->] (-4.5,0) -- (4.5,0) node[right] {$x$};
\draw[->] (0,-2) -- (0,4.5) node[above] {$y$};
\draw (0,0) -- (3,3) node[below right] {$Ax = b$};
\end{tikzpicture}
\end{center}

d) If $b, v, w$ are vectors in $\mathbb{R}^3$ and $\text{Span}\{b, v, w\} = \mathbb{R}^3$, is it possible that $b$ is in $\text{Span}\{v, w\}$? Justify your answer.
Problem 3. [10 points]

Johnny Rico believes that the secret to the universe can be found in the system of two linear equations in \( x \) and \( y \) given by

\[
\begin{align*}
  x - y &= h \\
  3x + hy &= 4
\end{align*}
\]

where \( h \) is a real number.

a) Find all values of \( h \) (if any) which make the system inconsistent. Briefly justify your answer.

b) Find all values of \( h \) (if any) which make the system have a unique solution. Briefly justify your answer.
Problem 4.

[11 points]

a) Find the parametric form of the general solution of the following system of equations. Clearly indicate which variables (if any) are free variables.

\[
\begin{align*}
    x_1 + 2x_2 + 2x_3 - x_4 &= 4 \\
    2x_1 + 4x_2 + x_3 - 2x_4 &= -1 \\
    -x_1 - 2x_2 - x_3 + x_4 &= -1
\end{align*}
\]

b) Write the set of solutions to

\[
\begin{align*}
    x_1 + 2x_2 + 2x_3 - x_4 &= 0 \\
    2x_1 + 4x_2 + x_3 - 2x_4 &= 0 \\
    -x_1 - 2x_2 - x_3 + x_4 &= 0
\end{align*}
\]

in parametric vector form.
The diagram below represents traffic in a city.

\[ \begin{align*} 
\text{Traffic flow (cars/hr)} \\
70 & \quad 90 \\
& \quad x_1 \\
& \quad x_3 \\
120 & \quad x_2 \\
60 & \quad 50 \\
& \quad 110 
\end{align*} \]

a) Write a system of three linear equations whose solution would give the values of \( x_1 \), \( x_2 \), and \( x_3 \). Do not solve it.

b) Write the system of equations as a vector equation. Do not solve it.
[Scratch work]