## Math 1553 Worksheet §6.4, 6.5

1. Answer yes, no, or maybe. Justify your answers. In each case, $A$ is a matrix whose entries are real numbers.
a) If $A$ is a $3 \times 3$ matrix with characteristic polynomial $-\lambda(\lambda-5)^{2}$, then the 5eigenspace is 2-dimensional.
b) If $A$ is an invertible $2 \times 2$ matrix, then $A$ is diagonalizable.
c) A $3 \times 3$ matrix $A$ can have a non-real complex eigenvalue with multiplicity 2 .
d) Suppose $A$ is a $7 \times 7$ matrix with four distinct eigenvalues. If one eigenspace has dimension 2 , while another eigenspace has dimension 3 , then $A$ must be diagonalizable.
2. $A=\left(\begin{array}{ccc}2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1\end{array}\right)$.
a) Find the eigenvalues of $A$, and find a basis for each eigenspace.
b) Is $A$ diagonalizable? If your answer is yes, find a diagonal matrix $D$ and an invertible matrix $C$ so that $A=C D C^{-1}$. If your answer is no, justify why $A$ is not diagonalizable.
