1. If $A$ is a $3 \times 5$ matrix and $B$ is a $3 \times 2$ matrix, which of the following are defined? Very briefly justify your answer.
   a) $A - B$
   b) $AB$
   c) $A^T B$
   d) $B^T A$
   e) $A^2$

2. True or false (justify your answer). Answer true if the statement is always true. Otherwise, answer false.
   a) Suppose $A$ and $B$ are matrices and the matrix product $AB$ is defined. Then each column of $AB$ must be a linear combination of the columns of $A$.
   b) If $A$ is a $3 \times 4$ matrix and $B$ is a $4 \times 2$ matrix, then the linear transformation $Z$ defined by $Z(x) = ABx$ has domain $\mathbb{R}^2$ and codomain $\mathbb{R}^3$.
   c) Suppose $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $U: \mathbb{R}^m \rightarrow \mathbb{R}^p$ are linear transformations and $U \circ T$ is onto. Then $U$ and $T$ must both be onto.
3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation clockwise by $60^\circ$. Let $U : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix \[
\begin{pmatrix}
-2 & 1 \\
1 & 0
\end{pmatrix}.
\]

a) Find the standard matrix for the composition $U \circ T$.

b) Find the standard matrix for the composition $T \circ U$.

c) Is rotating clockwise by $60^\circ$ and then performing $U$, the same as first performing $U$ and then rotating clockwise by $60^\circ$?