Supplemental problems: §1.2, §1.3

1. Is the matrix below in reduced row echelon form?
\[
\begin{pmatrix}
1 & 1 & 0 & -3 & 1 \\
0 & 0 & 1 & -1 & 5 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

2. Put an augmented matrix into reduced row echelon form to solve the system
\[
x_1 - 2x_2 - 9x_3 + x_4 = 3 \\
4x_2 + 8x_3 - 24x_4 = 4.
\]

3. a) Row reduce the following matrices to reduced row echelon form.

b) If these are augmented matrices for a linear system (with the last column being after the = sign), then which are inconsistent? Which have a unique solution?
\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
4 & 5 & 6 & 7 \\
6 & 7 & 8 & 9
\end{pmatrix}
\begin{pmatrix}
1 & 3 & 5 & 7 \\
3 & 5 & 7 & 9 \\
5 & 7 & 9 & 1
\end{pmatrix}
\begin{pmatrix}
3 & -4 & 2 & 0 \\
-8 & 12 & -4 & 0 \\
-6 & 8 & -1 & 0
\end{pmatrix}
\]

4. We can use linear algebra to find a polynomial that fits given data, in the same way that we found a circle through three specified points in the §1.1 Webwork.

Is there a degree-three polynomial \( P(x) \) whose graph passes through the points \((-2, 6), (−1, 4), (1, 6), \) and \((2, 22)\)? If so, how many degree-three polynomials have a graph through those four points? We answer this question in steps below.

a) If \( P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \) is a degree-three polynomial passing through the four points listed above, then \( P(−2) = 6, \ P(−1) = 4, \ P(1) = 6, \) and \( P(2) = 22. \) Write a system of four equations which we would solve to find \( a_0, \ a_1, \ a_2, \) and \( a_3. \)

b) Write the augmented matrix to represent this system and put it into reduced row-echelon form. Is the system consistent? How many solutions does it have?