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Math 1553 Quiz 1, Fall 2019 (10 points, 10 minutes)
Solutions

Show your work on problem 4 or you may receive little or no credit. You do not need to show work or justify your answers on problems 1 through 3.

1. (1 point each) For each statement, clearly circle TRUE or FALSE.

a) If a system of linear equations has more variables than equations, then the system must be consistent.   FALSE
This question was taken almost directly from Webwork. For example, consider the inconsistent system
$$x + y + z = 5$$
$$x + y + z = 6.$$

b) The equation $e^\pi x - 8y = z - 12$ is a linear equation in x, y, and z.
      TRUE
This question was basically taken from the first worksheet.

2. (3 points) Write a system of two linear equations in two variables x and y that has exactly one solution.

Solution.

Many examples possible, just take two non-parallel lines in the xy-plane, for example
$$x + y = 1$$
$$x - y = 2.$$

3. (1 point) Write one point (x, y, z) in $\mathbf{R}^3$ that satisfies
$$2x - y + z = 1.$$

Solution.

Many examples possible, for example $(0, 0, 1)$ since $2(0) - 0 + 1 = 1$. Also, $(1, 1, 0)$ since $2(1) - 1 + 0 = 1$.

4. (4 points) Find all values of h (if there are any) so that the following system of equations is inconsistent.

$$x - 3y = 7$$
$$2x + hy = 5$$

Solution.

Solution using algebra: We subtract twice the first equation from the second to get:

$$x - 3y = 7$$
$$(h + 6)y = -9$$

If $h + 6 = 0$, the second equation is $0 = -9$ which makes our system inconsistent.

If $h + 6 \neq 0$, then $y = -\dfrac{9}{h + 6}$ and we can use the first equation (back-substitition for example) to solve for x.

Therefore, there is exactly one value of h that makes the system inconsistent: $h = -6$.

Solution using geometry: The system will be inconsistent if and only if the lines $x - 3y = 7$ and $2x + hy = 5$ in the xy-plane are different parallel lines.

One way to solve it is the following: If we multiply the first equation by two, the system becomes

$$2x - 6y = 14$$
$$2x + hy = 5.$$

These lines are parallel when the y-coefficients are equal (so that both lines have the same slope), which is when $h = -6$. In this case, the left sides match but the right sides are different, so these two lines cannot be identical.

It is also fine if the student uses augmented matrices and a pivot argument, which is totally analogous to our solution using algebra.