Name:_____ Studio Section:_____

Math 1553 Quiz 2, Fall 2019 (10 points, 10 minutes)

Solutions

As always, RREF means "reduced row echelon form." Show your work on problem 2 or you may receive little or no credit.

1. (1 point each) For each statement, clearly circle TRUE or FALSE.

a) If an augmented matrix has a row of zeros in its RREF, then the corresponding system of linear equations has infinitely many solutions. FALSE.
The system might be inconsistent. Taken almost verbatim from Webwork.

b) The matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is in RREF. TRUE

c) The matrix $\left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ is in RREF. TRUE.
Keep in mind that the rightmost column is not a pivot column in this augmented matrix, so it is fine that we have the entry -2 in that column.

2. (7 points) Consider the following linear system of equations in x_1, x_2, and x_3.

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ -x_1 - 2x_2 - 4x_3 &= 1 \\ 3x_1 + 3x_2 + 3x_3 &= 9. \end{aligned}$$

a) Write the system as an augmented matrix and put it into RREF.

b) Solve the system. Write its general solution in parametric form and clearly indicate which variables (if any) are free.

Solution.

a)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ -1 & -2 & -4 & 1 \\ 3 & 3 & 3 & 9 \end{array}\right) \xrightarrow[R_3=R_3-3R_1]{R_2=R_2+R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2=-R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_1=R_1-R_2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

b) From the RREF in (a) we see the system is consistent, and x_3 is free since it has no pivot in its column.

$$x_1 = 7 + 2x_3, \qquad x_2 = -4 - 3x_3, \qquad x_3 = x_3 \quad (x_3 \text{ real})$$