Name:______________________________ Studio Section:__________

Math 1553 Combined Quiz 7, Fall 2019 (10 points, 10 minutes)

Solutions

No work or justification is required except on 3(b). Show your work in 3(b) or you may receive little or no credit.

These are solutions for the combined versions of quiz 7. As announced in class, some students took a slightly older version of the quiz than others, so we gave full credit for the differing problems and we only graded the overlap. Here, all four combined true / false questions are listed. Only the last one (the one marked here as "(d)") was graded for accuracy.

1. (2 points) Complete the following definition (be mathematically precise!):
Suppose A is an $n \times n$ matrix. A vector v in $\mathbf{R}^n$ is an *eigenvector* of A if...

v is not the zero vector and $Av = \lambda v$ for some scalar λ.

2. (1 point each) True or false. If the statement is *always* true, answer TRUE. Otherwise, circle FALSE.

a) If A and B are $n \times n$ matrices and $\det(A) = \det(B)$, then A and B have the same eigenvalues. TRUE $\boxed{\text{FALSE}}$

For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

b) If A is a 2×2 matrix and $A\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then $\lambda = 0$ is an eigenvalue of A.

$\boxed{\text{TRUE}}$ FALSE $Ax = 0$ has nontrivial solution $x = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

c) If A and B are $n \times n$ matrices with the same characteristic polynomial, then A and B must have the same eigenvalues. $\boxed{\text{TRUE}}$ FALSE
The char polys are the same and thus have the same roots (the eigenvalues).

d) Suppose A is a 2×2 matrix and v and w are nonzero vectors in $\mathbf{R}^2$. If $Av = 2v$ and $Aw = 3w$, then $\{v, w\}$ must be a basis for $\mathbf{R}^2$. $\boxed{\text{TRUE}}$ FALSE
Note v and w are lin. ind. since they correspond to different eigenvalues.

3. Suppose $A = \begin{pmatrix} 4 & -3 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{pmatrix}$.

a) Write the eigenvalues of A. (you do not need to show your work for this part)

Since A is upper-triangular its eigenvalues are its diagonal entries, namely $\lambda = 4$, $\lambda = 1$, and $\lambda = 3$.

b) For the largest eigenvalue λ of A, find a basis for the λ-eigenspace.

$$(A - 4I \mid 0) = \left(\begin{array}{ccc|c} 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \xrightarrow{RREF} \left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

Thus $x_1 = x_1$ (free) and $x_2 = x_3 = 0$. A basis for the 4-eigenspace is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$.