1. Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day. Today, Courage has 80 customers and Dexter has 130 customers. Each day:
   70% of Courage Soda’s customers keep drinking Courage Soda, while 30% switch to Dexter Soda.
   40% of Dexter Soda’s customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

   a) Write a stochastic matrix $A$ and a vector $x$ so that $Ax$ will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow. You do not need to compute $Ax$.

   b) A quick computation shows that the 1-eigenspace for this positive stochastic matrix $A$ is spanned by \( \begin{pmatrix} 2 \\ 1 \end{pmatrix} \).

   Find the steady-state vector for $A$. In the long run, roughly how many daily customers will Courage Soda have?
2. Let $W$ be the set of all vectors in $\mathbb{R}^3$ of the form $(x, x - y, y)$ where $x$ and $y$ are real numbers.
   
a) Find a basis for $W^\perp$.

b) Find the matrix $B$ for orthogonal projection onto $W$. 