Supplemental problems: §1.2, §1.3

1. Is the matrix below in reduced row echelon form?
   \[
   \begin{pmatrix}
   1 & 1 & 0 & -3 & 1 \\
   0 & 0 & 1 & -1 & 5 \\
   0 & 0 & 0 & 0 & 0
   \end{pmatrix}
   \]

2. Put an augmented matrix into reduced row echelon form to solve the system
   \[
   x_1 - 2x_2 - 9x_3 + x_4 = 3 \\
   4x_2 + 8x_3 - 24x_4 = 4.
   \]

3. a) Row reduce the following matrices to reduced row echelon form.
    
    b) If these are augmented matrices for a linear system (with the last column being after the = sign), then which are inconsistent? Which have a unique solution?
   \[
   \begin{pmatrix}
   1 & 2 & 3 & 4 \\
   4 & 5 & 6 & 7 \\
   6 & 7 & 8 & 9
   \end{pmatrix} \quad \begin{pmatrix}
   1 & 3 & 5 & 7 \\
   3 & 5 & 7 & 9 \\
   5 & 7 & 9 & 1
   \end{pmatrix} \quad \begin{pmatrix}
   3 & -4 & 2 & 0 \\
   -8 & 12 & -4 & 0 \\
   -6 & 8 & -1 & 0
   \end{pmatrix}
   \]

4. We can use linear algebra to find a polynomial that fits given data, in the same way that we found a circle through three specified points in the §1.2 Webwork.
   Is there a degree-three polynomial \( P(x) \) whose graph passes through the points \((-2, 6), (-1, 4), (1, 6), \) and \((2, 22)\)? If so, how many degree-three polynomials have a graph through those four points? We answer this question in steps below.
   
   a) If \( P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \) is a degree-three polynomial passing through the four points listed above, then \( P(-2) = 6, \ P(-1) = 4, \ P(1) = 6, \) and \( P(2) = 22. \) Write a system of four equations which we would solve to find \( a_0, \ a_1, a_2, \) and \( a_3. \)
   
   b) Write the augmented matrix to represent this system and put it into reduced row-echelon form. Is the system consistent? How many solutions does it have?