Supplemental problems: §5.5

1. a) If $A$ is the matrix that implements rotation by $143\degree$ in $\mathbb{R}^2$, then $A$ has no real eigenvalues.

   b) A $3 \times 3$ matrix can have eigenvalues $3$, $5$, and $2 + i$.

   c) If $v = \begin{pmatrix} 2 + i \\ 1 \end{pmatrix}$ is an eigenvector of $A$ corresponding to the eigenvalue $\lambda = 1 - i$, then $w = \begin{pmatrix} 2i - 1 \\ i \end{pmatrix}$ is an eigenvector of $A$ corresponding to the eigenvalue $\lambda = 1 - i$.

2. Consider the matrix

   
   \[
   A = \begin{pmatrix}
   3\sqrt{3} - 1 & -5\sqrt{3} \\
   2\sqrt{3} & -3\sqrt{3} - 1
   \end{pmatrix}
   \]

   a) Find both complex eigenvalues of $A$.

   b) Find an eigenvector corresponding to each eigenvalue.

3. This one is just for fun! It demonstrates, by example, that a matrix can have a mix of real and non-real complex eigenvalues, and that we can find a basis for each eigenspace in the usual fashion, even if it takes more work than usual.

   Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$. Find all eigenvalues of $A$. For each eigenvalue of $A$, find a corresponding eigenvector.
Supplemental problems: §5.6

1. Suppose the internet has four pages in the following manner. Arrows represent links from one page towards another. For example, page 1 links to page 4 but not vice versa.

![Diagram of internet structure]

a) Write the importance matrix for this internet.

b) Assume there is no damping factor, so the importance matrix is the Google matrix. The 1-eigenspace is spanned by \[
\begin{pmatrix}
\frac{3}{4} \\
\frac{3}{4} \\
\frac{3}{4} \\
1
\end{pmatrix}
\]. Find the steady-state vector for the Google matrix. What page has the highest rank?

2. The companies X, Y, and Z fight for customers. This year, company X has 40 customers, Company Y has 15 customers, and Z has 20 customers. Each year, the following changes occur:
   - X keeps 75% of its customers, while losing 15% to Y and 10% to Z.
   - Y keeps 60% of its customers, while losing 5% to X and 35% to Z.
   - Z keeps 65% of its customers, while losing 15% to X and 20% to Y.

Write a stochastic matrix \( A \) and a vector \( x \) so that \( Ax \) will give the number of customers for firms X, Y, and Z (respectively) after one year. You do not need to compute \( Ax \).