Math 1553 Quiz 2, Spring 2020  
Jankowski, Lecture A1-A3 (8:00 AM)

Solutions

Show your work on problem 3 or you may receive little or no credit. You do not need to show work or justify your answers on problems 1 and 2.

1. (1 point each) Which of the following matrices are in RREF (reduced row echelon form)? Clearly circle all that apply.
   
   a) \[
   \begin{pmatrix}
   1 & 2 & 0 \\
   0 & 0 & 1 \\
   \end{pmatrix}
   \begin{pmatrix}
   7 \\
   -1 \\
   \end{pmatrix}
   \]
   
   b) \[
   \begin{pmatrix}
   0 & 0 & 1 \\
   0 & 0 & 0 \\
   \end{pmatrix}
   \]

   Both (a) and (b) are in RREF.

2. (1 point each) Consider the augmented matrices below. For each matrix, determine whether the corresponding system of linear equations has no solution, exactly one solution, or infinitely many solutions. Clearly circle your answer.

   a) \[
   \begin{pmatrix}
   1 & -4 & 0 & 0 \\
   0 & 0 & 0 & 1 \\
   \end{pmatrix}
   \begin{pmatrix}
   0 \\
   0 \\
   \end{pmatrix}
   \]
   
   No solution \quad Exactly 1 solution \quad Infinitely many solutions

   b) \[
   \begin{pmatrix}
   1 & 0 & 1 \\
   0 & 1 & -2 \\
   0 & 0 & 0 \\
   \end{pmatrix}
   \]
   
   No solution \quad Exactly 1 solution \quad Infinitely many solutions

Turn to the back side for problem 3!
3. (6 points) Consider the following system of linear equations:

\[
\begin{align*}
    x_1 + x_2 - x_3 &= 2 \\
    2x_1 + 2x_2 - 5x_3 &= 13 \\
    -x_1 - x_2 + x_3 &= -2.
\end{align*}
\]

a) (3 points) Write the augmented matrix corresponding to the system, and put the matrix into RREF.

\[
\begin{pmatrix}
    1 & 1 & -1 & 2 \\
    2 & 2 & -5 & 13 \\
    -1 & -1 & 1 & -2
\end{pmatrix}
\]

\[
\begin{pmatrix}
    1 & 1 & -1 & 2 \\
    2 & 2 & -5 & 13 \\
    -1 & -1 & 1 & -2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    1 & 1 & -1 & 2 \\
    0 & 0 & -3 & 9 \\
    0 & 0 & 0 & 0
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    1 & 1 & 0 & -1 \\
    0 & 0 & 1 & -3 \\
    0 & 0 & 0 & 0
\end{pmatrix}
\]

b) (3 points) Write the solution set to the system of equations in parametric form.

We see \(x_2\) is free, and the equations from the RREF are

\[
\begin{align*}
    x_1 + x_2 &= -1, \\
    x_2 &= x_2 \text{ (}x_2\text{ real)}, \\
    x_3 &= -3.
\end{align*}
\]

Thus,

\[
\begin{align*}
    x_1 &= -1 - x_2, \\
    x_2 &= x_2 \text{ (}x_2\text{ real)}, \\
    x_3 &= -3.
\end{align*}
\]