Math 1553 Quiz 3, Spring 2020  (10 points, 10 minutes)
Jankowski, Lecture A1-A3 (8:00 AM)

Solutions

Unless specified otherwise, show your work or you may receive little or no credit, even if your answer is correct.

1.  a) (2 points) Complete the following definition (be mathematically precise!):
Let \( v_1, v_2, \ldots, v_p \) be vectors in \( \mathbb{R}^n \). The span of \( v_1, v_2, \ldots, v_p \), which we denote by \( \text{Span}\{v_1, \ldots, v_p\} \), is...

   Solution: In set-builder notation,
   \[
   \text{Span}\{v_1, v_2, \ldots, v_p\} = \left\{ x_1 v_1 + \cdots + x_p v_p \mid x_1, \ldots, x_p \text{ real} \right\}.
   \]
   (the dividing bar could be a colon if desired, and “real” could be “scalar” here; that is just a matter of notation)

   Alternatively, the student could state that \( \text{Span}\{v_1, v_2, \ldots, v_p\} \) is the set of all linear combinations of \( v_1, v_2, \ldots, v_p \).

   b) (1 point) Determine whether the following statement is true or false. Clearly circle your answer (you do not need to show work or justify your answer):
   If \( v_1 \) and \( v_2 \) are nonzero vectors in \( \mathbb{R}^2 \), then \( \text{Span}\{v_1, v_2\} = \mathbb{R}^2 \).
   TRUE   FALSE
   For example, take \( v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( v_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \).

2.  (3 points) Let \( v \) and \( w \) be the vectors labeled below. Draw \( w - 2v \) as an arrow.

   Solution: We use the tip of \( w \) and take two steps backward in the direction of \( v \).

   \[ \text{turn over for problem 3!} \]
3. (4 points) Find all real numbers \( h \) (if there are any) so that the following vector equation will have exactly one solution:

\[
x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}.
\]

**Solution:** We set up the vector equation’s corresponding augmented matrix:

\[
\begin{pmatrix} 1 & -2 & 3 \\ 3 & h & 9 \end{pmatrix} \xrightarrow{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & -2 & 3 \\ 0 & h + 6 & 0 \end{pmatrix}.
\]

The system must be consistent because the rightmost column of the augmented matrix does not have a pivot.

The system will have exactly one solution precisely when there is a pivot in both columns to the left of the augment, hence \( h + 6 \neq 0 \), thus \( h \neq -6 \).