1. a) (2 points) Complete the following definition (be mathematically precise!):
Let \( w, v_1, v_2, \ldots, v_p \) be vectors in \( \mathbb{R}^n \). We say \( w \) is a linear combination of \( v_1, v_2, \ldots, v_p \) if...

\[
w = c_1 v_1 + \cdots + c_p v_p \quad \text{for some scalars } c_1, \ldots, c_p.
\]

It is fine if the student uses different symbols than “\( c_i \)” or writes “real numbers” rather than “scalars” in the definition.

b) (1 point) Determine whether the following statement is true or false. Clearly circle your answer (you do not need to show work or justify your answer):
If \( v_1 \) and \( v_2 \) are vectors in \( \mathbb{R}^3 \) and \( v_1 \neq v_2 \), then \( \text{Span}\{v_1, v_2\} \) must be a plane.

TRUE \hspace{1cm} FALSE \quad \text{For example, } v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } v_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.

2. (3 points) Let \( v \) and \( w \) be the vectors labeled below. Draw \( v - 2w \) as an arrow.

Solution: We use the tip of \( v \) and take two steps backward in the direction of \( w \).
3. (4 points) Find all real numbers \( h \) (if there are any) so that the following vector equation has infinitely many solutions:

\[
\begin{pmatrix} x_1 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ h \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.
\]

We set up the augmented matrix and row-reduce:

\[
\begin{pmatrix} 1 & -1 & 2 \\ 2 & h & 4 \end{pmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 1 & -1 & 2 \\ 0 & h + 2 & 0 \end{pmatrix}.
\]

From the above, we see the system is guaranteed to be consistent, and it will have infinitely many solutions if the second column fails to have a pivot. Thus \( h + 2 = 0 \), so \( h = -2 \).