Math 1553 Quiz 4, Spring 2020  (10 points, 10 minutes)
Jankowski, Lecture A1-A3 (8:00 AM)

Solutions

On this quiz, you do not need to show your work or justify your answers, since every question is either TRUE/FALSE or short answer.

1. True or false. If the statement is always true, circle TRUE. Otherwise, circle FALSE.

   a) If $W$ is a 2-dimensional subspace of $\mathbb{R}^4$ containing $v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, then $\{v_1, v_2\}$ must be a basis for $W$.  TRUE  FALSE

   b) There is a $3 \times 5$ matrix $A$ satisfying $\text{nullity}(A) = 1$.  TRUE  FALSE

2. (4 points) Consider the matrix $A$ and its RREF, which are given below:

   \[
   A = \begin{pmatrix}
   1 & -3 & 2 \\
   2 & -6 & 0 \\
   4 & -12 & 3 \\
   1 & -3 & 1 \\
   \end{pmatrix} \rightarrow \begin{pmatrix}
   1 & -3 & 0 \\
   0 & 0 & 1 \\
   0 & 0 & 0 \\
   0 & 0 & 0 \\
   \end{pmatrix},
   \]

   Define a matrix transformation $T$ by $T(x) = Ax$.

   a) What is the domain of $T$? $\mathbb{R}^3$

   b) What is the codomain of $T$? $\mathbb{R}^4$

   c) Write a basis for the range of $T$.

      The pivot columns of $A$ form a basis for range($T$): $\left\{ \begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \\ 1 \end{pmatrix} \right\}$. In fact, if $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$, then $\{v_2, v_3\}$ would also be a basis for range($T$), but $\{v_1, v_2\}$ would not.
3. (4 points) Match each matrix with its corresponding matrix transformation from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \), which is given by some roman numeral from (i) through (viii).

\[
A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \text{corresponds to} \quad \text{(ii) Reflection across the line} \; y = -x.
\]

\[
B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{corresponds to} \quad \text{(v) Rotation by 90° counterclockwise.}
\]

\[
C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{corresponds to} \quad \text{(viii) Reflection across the y-axis.}
\]

\[
D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{corresponds to} \quad \text{(iv) The identity trans.} \; T(x, y) = (x, y).
\]

(i) Reflection across the line \( y = x \).
(ii) Reflection across the line \( y = -x \).
(iii) Projection onto the \( x \)-axis given by \( T(x, y) = (x, 0) \)
(iv) The identity transformation given by \( T(x, y) = (x, y) \).
(v) Rotation by 90° counterclockwise.
(vi) Rotation by 90° clockwise.
(vii) Reflection across the \( x \)-axis.
(viii) Reflection across the \( y \)-axis.