Math 1553 Quiz 4, Spring 2020  (10 points, 10 minutes)
Jankowski, Lecture C1-C4 (11:15 AM)

Solutions

On this quiz, you do not need to show your work or justify your answers, since every question is either TRUE/FALSE or short answer.

1. True or false. If the statement is always true, circle TRUE. Otherwise, circle FALSE.
   a) If $W$ is a 2-dimensional subspace of $\mathbb{R}^5$, then every basis of $W$ must consist of exactly 2 vectors.  \text{TRUE} \quad \text{FALSE}
   
b) If $A$ is a $10 \times 15$ matrix and the set of solutions to $Ax = 0$ has dimension 8, then $\text{Col}(A)$ is a 7-dimensional subspace of $\mathbb{R}^{10}$. \text{TRUE} \quad \text{FALSE}

2. (4 points) Consider the matrix $A$ and its RREF, which are given below:
   
   $$A = \begin{pmatrix}
   1 & -2 & -1 & 2 \\
   0 & 0 & -2 & 4 \\
   2 & -4 & -4 & 8
   \end{pmatrix} \rightarrow \begin{pmatrix}
   1 & -2 & 0 & 0 \\
   0 & 0 & 1 & -2 \\
   0 & 0 & 0 & 0
   \end{pmatrix}.
   $$

   Define a matrix transformation $T$ by $T(x) = Ax$.
   
   a) What is the domain of $T$? $\mathbb{R}^4$
   
   b) What is the codomain of $T$? $\mathbb{R}^3$
   
   c) Write a basis for the range of $T$.
      
      The pivot columns of $A$ form a basis for $\text{range}(T)$: \( \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -4 \end{pmatrix} \right\} \). In fact, if $A = \begin{pmatrix} v_1 & v_2 & v_3 & v_4 \end{pmatrix}$, then any choice of two columns of $A$ will be a basis for $\text{range}(T)$ except for $\{v_1, v_2\}$ or $\{v_3, v_4\}$.

   \text{turn over for problem 3!}
3. (4 points) Match each matrix with its corresponding matrix transformation from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \), which is given by some roman numeral from (i) through (viii).

\[
A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{corresponds to} \quad (v) \text{ Rotation by } 90^\circ \text{ counterclockwise.}
\]

\[
B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{corresponds to} \quad (i) \text{ Reflection across the line } y = x.
\]

\[
C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{corresponds to} \quad (vii) \text{ Reflection across the x-axis.}
\]

\[
D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{corresponds to} \quad (iv) \text{ The identity trans. } T(x, y) = (x, y).
\]

(i) Reflection across the line \( y = x \).
(ii) Reflection across the line \( y = -x \).
(iii) Projection onto the x-axis given by \( T(x, y) = (x, 0) \)
(iv) The identity transformation given by \( T(x, y) = (x, y) \).
(v) Rotation by \( 90^\circ \) counterclockwise.
(vi) Rotation by \( 90^\circ \) clockwise.
(vii) Reflection across the x-axis.
(viii) Reflection across the y-axis.