Math 1553 Conceptual question list §§4.1-5.6

Worksheet 8 (4.1-5.1)

1. Let A be an $n \times n$ matrix.

a) If $\det(A) = 1$ and c is a scalar, what is $\det(cA)$?

b) Using cofactor expansion, explain why $\det(A) = 0$ if A has adjacent identical columns.

2. In this problem, you need not explain your answers; just circle the correct one(s). Let A be an $n \times n$ matrix.

a) Which **one** of the following statements is correct?

1. An eigenvector of A is a vector v such that $Av = \lambda v$ for a nonzero scalar λ.
2. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a scalar λ.
3. An eigenvector of A is a nonzero scalar λ such that $Av = \lambda v$ for some vector v.
4. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a nonzero scalar λ.

b) Which **one** of the following statements is **not** correct?

1. An eigenvalue of A is a scalar λ such that $A - \lambda I$ is not invertible.
2. An eigenvalue of A is a scalar λ such that $(A - \lambda I)v = 0$ has a solution.

3. An eigenvalue of A is a scalar λ such that $Av = \lambda v$ for a nonzero vector v.
4. An eigenvalue of A is a scalar λ such that $\det(A - \lambda I) = 0$.

3. True or false: If v_1 and v_2 are linearly independent eigenvectors of an $n \times n$ matrix A, then they must correspond to different eigenvalues.

4. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T =$ projection onto the xz-plane in $\mathbf{R}^3$.

b) $T =$ reflection over $y = 2x$ in $\mathbf{R}^2$.

supplemental (4.1-5.1)

1. **a)** Is there a real 2×2 matrix A that satisfies $A^4 = -I_2$? Either write such an A, or show that no such A exists.
(hint: think geometrically! The matrix $-I_2$ represents rotation by π radians).

b) Is there a real 3×3 matrix A that satisfies $A^4 = -I_3$? Either write such an A, or show that no such A exists.

2. True or false. Answer true if the statement is always true. Otherwise, answer false.

a) If A and B are $n \times n$ matrices and A is row equivalent to B, then A and B have the same eigenvalues.

b) If A is an $n \times n$ matrix and its eigenvectors form a basis for $\mathbf{R}^n$, then A is invertible.

c) If 0 is an eigenvalue of the $n \times n$ matrix A, then $\text{rank}(A) < n$.

d) The diagonal entries of an $n \times n$ matrix A are its eigenvalues.

e) If A is invertible and 2 is an eigenvalue of A, then $\dfrac{1}{2}$ is an eigenvalue of A^{-1}.

f) If $\det(A) = 0$, then 0 is an eigenvalue of A.

g) If v and w are eigenvectors of a square matrix A, then so is $v + w$.

Worksheet 9 (5.1-5.4)

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that A is an $n \times n$ matrix.

a) The entries on the main diagonal of A are the eigenvalues of A.

b) The number λ is an eigenvalue of A if and only if there is a nonzero solution to the equation $(A - \lambda I)x = 0$.

c) To find the eigenvectors of A, we reduce the matrix A to row echelon form.

d) If A is invertible and 2 is an eigenvalue of A, then $\frac{1}{2}$ is an eigenvalue of A^{-1}.

e) If Nul(A) has dimension at least 1, then Nul(A) is the eigenspace of A corresponding to the eigenvalue 0.

2. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.

3. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

a) Suppose $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$. Then the characteristic polynomial of A is

$$\det(A - \lambda I) = (3 - \lambda)(1 - \lambda)(7 - \lambda).$$

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda-5)^2$, then the 5-eigenspace is 2-dimensional.

c) If A is an invertible 2×2 matrix, then A is diagonalizable.

supplemental (5.1-5.4)

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false.

a) If A and B are $n \times n$ matrices with the same eigenvectors, then A and B have the same characteristic polynomial.

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda^3 + \lambda^2 + \lambda$, then A is invertible.

2. True or false. Answer true if the statement is always true. Otherwise, answer false.

a) If A is an invertible matrix and A is diagonalizable, then A^{-1} is diagonalizable.

b) A diagonalizable $n \times n$ matrix admits n linearly independent eigenvectors.

c) If A is diagonalizable, then A has n distinct eigenvalues.

3. Give examples of 2×2 matrices with the following properties. Justify your answers.

a) A matrix A which is invertible and diagonalizable.

b) A matrix B which is invertible but not diagonalizable.

c) A matrix C which is not invertible but is diagonalizable.

d) A matrix D which is neither invertible nor diagonalizable.

Worksheet 10 (5.4-5.6)

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. If not explicitly stated, assume A, B are $n \times n$ matrices.

a) If A is diagonalizable and B is row equivalent to A, then B is also diagonalizable.

b) If A and B are diagonalizable, then AB is diagonalizable.

c) A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

d) If A is the 3×3 the matrix for the orthogonal projection of vectors in $\mathbf{R}^3$ onto the plane $x + y + z = 0$, then A is diagonalizable.

supplemental (5.4-5.6)

1. True/False

a) If A is the matrix that implements rotation by 143° in $\mathbf{R}^2$, then A has no real eigenvalues.

b) A 3×3 matrix can have eigenvalues $3, 5$, and $2+i$.

c) If $v = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$, then $w = \begin{pmatrix} 2i-1 \\ i \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$.