

Math 1553 Worksheet §2.6, 2.7, 2.9, 3.1, 3.2

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×10 matrix with 2 pivots in its RREF, then $\dim(\text{Nul}A) = 8$ and $\text{rank}(A) = 2$.

TRUE **FALSE**

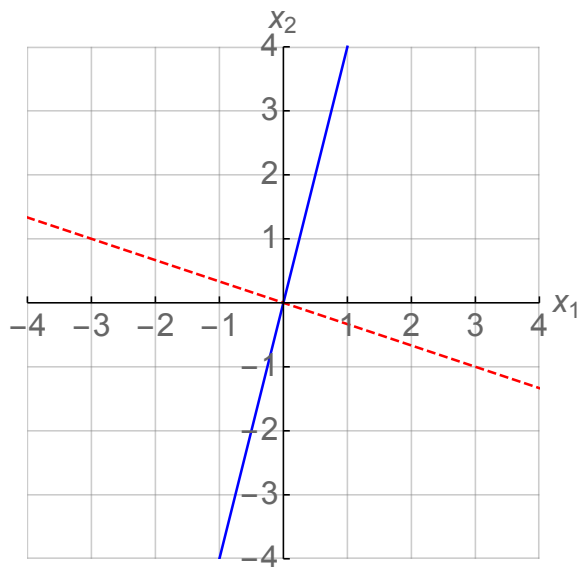
b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the transformation $T(x) = Ax$ is onto.

TRUE **FALSE**

c) If $\{a, b, c\}$ is a basis of a linear space V , then $\{a, a + b, b + c\}$ is a basis of V as well.

TRUE **FALSE**

2. Write a matrix A so that $\text{Col}(A)$ is the solid blue line and $\text{Nul}(A)$ is the dotted red line drawn below.



3. Let $A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$, and let T be the matrix transformation associated to A , so $T(x) = Ax$.

a) What is the domain of T ? What is the codomain of T ? Give an example of a vector in the range of T .

b) The RREF of A is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Is there a vector in the codomain of T which is not in the range of T ? Justify your answer.

c) Is T one-to-one? Is T onto? Justify your answer.

4. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the transformation is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by 32° in \mathbf{R}^2 .

b) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.