Math 1553 Worksheet §5.4, 5.5, 5.6

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. If not explicitly stated, assume A, B are $n \times n$ matrices.

   a) If A is diagonalizable and B is row equivalent to A, then B is also diagonalizable.

   b) If A and B are diagonalizable, then AB is diagonalizable.

   c) A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

   d) If A is the 3×3 the matrix for the orthogonal projection of vectors in $\mathbf{R}^3$ onto the plane $x + y + z = 0$, then A is diagonalizable.

2. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$, and let $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. What happens to $A^n x$ as n gets very large?

3. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$. Find all eigenvalues of A. For each eigenvalue, find an associated eigenvector.

4. Robert G. Durant's video game offers participants the chance to play as one of three characters: Archer, Barbarian, or Cleric. The game has 72 million customers.

In 2019:
Archer is played by 22 million customers.
Barbarian is played by 36 million customers.
Cleric is played by 14 million customers.

One year later, in 2020:
  - 50% of the people who started with the Archer still play with the Archer, while 30% have switched to Barbarian and 20% have switched to Cleric.
  - 60% of the customers who stared with the Barbarian still play with the Barbarian, while 10% have switched to Archer and 30% have switched to Cleric.
  - 70% of the customers who stared with the Cleric still play with the Cleric, while 10% have switched to Archer and 20% have switched to Barbarian.

  a) Write down the stochastic matrix A which represents the change in each character's popularity from 2019 to 2020, and use it to find the number of people who played with each character in 2020.

  b) Suppose the trend continues each year. In the distant future, what will be the most popular character?

  You may use the fact that the 1-eigenspace of A is spanned by $\begin{pmatrix} 6 \\ 13 \\ 17 \end{pmatrix}$.