Math 1553 Worksheet §2.1, §2.2, §2.3

1. **a)** Write a set of three vectors whose span is a point in $\mathbf{R}^3$.

b) Write a set of three different vectors whose span is a line in $\mathbf{R}^3$.

c) Write a set of three different vectors whose span is a plane in $\mathbf{R}^3$.

d) In each of the above questions, if you use the three vectors form a matrix A, how many pivots does A have?

2. Consider the system of linear equations

$$\begin{aligned} x + 2y &= 7 \\ 2x + y &= -2 \\ -x - y &= 4. \end{aligned}$$

Question: Does this system have a solution? If so, what is the solution set?

a) Formulate this question as an augmented matrix.

b) Formulate this question as a vector equation.

c) Formulate this question into a matrix equation $Av = b$.

d) What does this mean in terms of spans?

e) Answer the question using the interactive demo.

f) Answer the question using row reduction.

3. Catherine Halsey has challenged you to find a hidden treasure, located at some point (a, b, c). She has honestly guaranteed you that the treasure can be found by starting at the origin and taking steps in directions given by

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} \qquad v_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}.$$

By decoding Catherine's message, you have discovered that the first and second coordinates of the treasure's location are (in order) -4 and 3.

a) What is the treasure's full location?

b) Give instructions for how to find the treasure by only moving in the directions given by v_1, v_2, and v_3.

4. True or false. If the statement is *always* true, answer True. Otherwise, answer False. In parts (a) and (b), A is an $m \times n$ matrix and b is a vector in $\mathbf{R}^m$.

a) If b is in the span of the columns of A, the matrix equation $Ax = b$ is consistent.

b) A does not have a pivot in every column if $Ax = b$ is inconsistent.

c) If A is a 4×3 matrix, then the equation $Ax = b$ is inconsistent for some b in $\mathbb{R}^4$.