Supplemental problems: Chapter 6

1. True or false. If the statement is always true, answer true. Otherwise, answer false. Justify your answer.

a) Suppose $W = \mathrm{Span}\{w\}$ for some vector $w \neq 0$, and suppose v is a vector orthogonal to w. Then the orthogonal projection of v onto W is the zero vector.

b) Suppose W is a subspace of $\mathbf{R}^n$ and x is a vector in $\mathbf{R}^n$. If x is not in W, then $x - x_W$ is not zero.

c) Suppose W is a subspace of $\mathbf{R}^n$ and x is in both W and $W^\perp$. Then $x = 0$.

d) Suppose $\widehat{x}$ is a least squares solution to $Ax = b$. Then $\widehat{x}$ is the closest vector to b in the column space of A.

2. Let $W = \mathrm{Span}\{v_1, v_2\}$, where $v_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

a) Find the closest point w in W to $x = \begin{pmatrix} 0 \\ 14 \\ -4 \end{pmatrix}$.

b) Find the distance from w to $\begin{pmatrix} 0 \\ 14 \\ -4 \end{pmatrix}$.

c) Find the standard matrix for the orthogonal projection onto $\mathrm{Span}\{v_1\}$.

d) Find the standard matrix for the orthogonal projection onto W.

3. Find the least-squares line $y = Mx + B$ that approximates the data points

$$(-2, -11), \quad (0, -2), \quad (4, 2).$$