MATH 1553, FALL 2023
SAMPLE MIDTERM 3A: COVERS 3.5 - 5.5

Name		**GT ID**	

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. We recommend completing the practice exam in 75 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to section §3.5 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to §§3.5 through 5.5.

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Problem 1.

For each statement, answer TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) Suppose S is a rectangle in $\mathbf{R}^2$ with area 2, and let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the matrix transformation

$$T(x) = \begin{pmatrix} 1 & 0 \\ 15 & 3 \end{pmatrix} x.$$

Then the area of $T(S)$ is 6.

TRUE FALSE

b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then A must be invertible.

TRUE FALSE

c) There is an $n \times n$ matrix A so that the zero vector is an eigenvector of A.

TRUE FALSE

d) Let $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. Then $\lambda = 0$ is an eigenvalue of A.

TRUE FALSE

e) Let A be the 2×2 matrix that rotates vectors in $\mathbf{R}^2$ by 65 degrees counterclockwise. Then A has no real eigenvalues. TRUE FALSE

Problem 2.

Parts (a), (b), (c), and (d) are unrelated. On (a) and (b), you do not need to show your work, and there is no partial credit. Show your work on (c) and (d).

a) (2 points) Suppose $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 7$. Find $\det\begin{pmatrix} d & e & f \\ a & b & c \\ -2a+g & -2b+h & -2c+i \end{pmatrix}$.

Clearly circle the correct answer below.

(i) 7 (ii) -7 (iii) 14 (iv) -14

(v) 56 (vi) -56 (vii) not enough information (viii) none of these

b) (2 points) Let $A = \begin{pmatrix} 1 & -4 \\ 3 & 15 \end{pmatrix}$. What is A^{-1}? Select the correct choice below.

(i) $A^{-1} = \frac{1}{3}\begin{pmatrix} 15 & -4 \\ 3 & 1 \end{pmatrix}$ (ii) $A^{-1} = \frac{1}{3}\begin{pmatrix} 15 & 4 \\ -3 & 1 \end{pmatrix}$ (iii) $A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 4 \\ -3 & 15 \end{pmatrix}$

(iv) $A^{-1} = \frac{1}{27}\begin{pmatrix} 15 & -4 \\ 3 & 1 \end{pmatrix}$ (v) $A^{-1} = \frac{1}{27}\begin{pmatrix} 15 & 4 \\ -3 & 1 \end{pmatrix}$ (vi) $A^{-1} = \frac{1}{27}\begin{pmatrix} 1 & 4 \\ -3 & 15 \end{pmatrix}$

c) (3 points) Find the area of the triangle with vertices $(1,2)$, $(2,3)$, and $(4,-5)$.

d) (3 points) Find all values of a so that $\det\begin{pmatrix} 1 & -3 & 0 \\ 1 & a & 0 \\ a & 0 & a \end{pmatrix} = 0$.

Problem 3.

Parts (a), (b), and (c) are unrelated. You do not need to show your work on this page, and there is no partial credit.

a) (3 points) Let A and B be 3×3 matrices satisfying $\det(A) = 2$ and $\det(B) = -3$. Which of the following must be true? Clearly circle all that apply.

(i) $\det(A+B) = \det(A) + \det(B)$.

(ii) $\det(A^T B^{-1}) = -2/3$.

(iii) $\det(-2A) = -16$.

b) (4 points) Suppose A is an $n \times n$ matrix. Which of the following conditions guarantee that $\lambda = 4$ is an eigenvalue of A? Clearly circle all that apply.

(i) The equation $(A - 4I)x = 0$ has infinitely many solutions.

(ii) There is a nonzero vector x in $\mathbf{R}^n$ so that the set $\{x, Ax\}$ is linearly dependent.

(iii) There is a non-trivial solution to the equation $Ax = 4x$.

(iv) $\mathrm{Nul}(A - 4I) = \{0\}$.

c) (3 points) Suppose A is a 3×3 matrix with characteristic polynomial

$$\det(A - \lambda I) = -\lambda(\lambda + 1)^2.$$

Which of the following statements are true? Clearly circle all that apply.

(i) The eigenvalues of A are -1 and 0.

(ii) A cannot be diagonalizable.

(iii) The null space of A must be 1-dimensional.

Problem 4.

Parts (a), (b), and (c) are unrelated.

a) Let A be the 3×3 matrix for projection onto the xy-plane in $\mathbf{R}^3$.
(i) (2 points) What are the eigenvalues of A?

(ii) (1 point) Is A diagonalizable? YES NO

b) (4 points) Let A be the 2×2 matrix that reflects vectors across the line $y = x$. Fill in the blanks below.

One eigenvalue of A is $\lambda_1 =$ ______ and an eigenvector for λ_1 is $v_1 = \begin{pmatrix} \\ \\ \end{pmatrix}$.

The other eigenvalue of A is $\lambda_2 =$ ______ and an eigenvector for λ_2 is $v_2 = \begin{pmatrix} \\ \\ \end{pmatrix}$.

c) (3 points) Which of the following statements must be true? Clearly circle all that apply.

(i) If A and B are invertible $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

(ii) An $n \times n$ matrix A is not invertible if one of its columns is a linear combination of its other columns.

(iii) If an $n \times n$ matrix A is invertible, then its reduced row echelon form is I_n (the $n \times n$ identity matrix).

Problem 5.

Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.

For this problem, let $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 4 & 4 \\ 0 & 0 & 3 \end{pmatrix}$.

a) (2 points) Find the eigenvalues of A. You do not need to show your work on this part.

b) (5 points) For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) (3 points) The matrix A is diagonalizable. Write a 3×3 matrix C and a 3×3 diagonal matrix D so that $A = CDC^{-1}$. Enter your answer below.

$$C = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix} \qquad D = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

Problem 6.

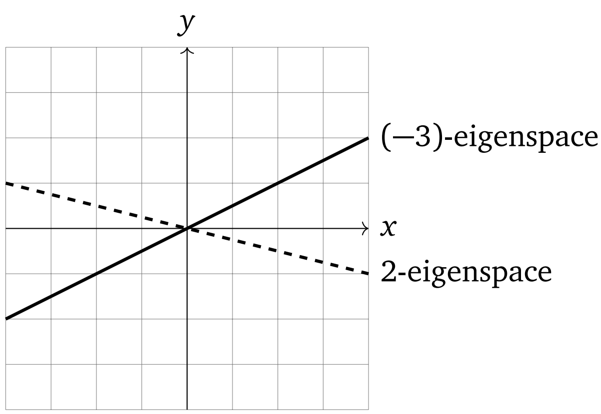
Free response. Fully simplify your answers. Parts (a) and (b) are unrelated. Show your work! A correct answer without sufficient work may receive little or no credit.

a) Let $A = \begin{pmatrix} 1 & -5 \\ 2 & 3 \end{pmatrix}$.

(i) (3 points) Find the complex eigenvalues of A.

(ii) (3 pts) For the eigenvalue with positive imaginary part, find an eigenvector v.

b) (4 points) Let A be the 2×2 matrix whose (-3)-eigenspace is the **solid** line below and whose 2-eigenspace is the **dashed** line below.



Find $A\begin{pmatrix} 6 \\ 0 \end{pmatrix}$. Enter your answer here: $A\begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$

Problem 7.

Free response. Show your work! A correct answer without sufficient work may receive little or no credit. Parts (a), (b), and (c) are unrelated.

a) (3 points) Write a 2×2 matrix A that satisfies both of the following conditions:
(i) A is not diagonalizable
(ii) A has exactly one real eigenvalue.
Justify why your matrix A satisfies both conditions.

b) (4 points) Find all values of c so that $\lambda = 2$ is an eigenvalue of $\begin{pmatrix} 3 & c \\ 2 & 1 \end{pmatrix}$.

c) (3 points) Find $\det \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & -1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 6 & 0 & 1 \end{pmatrix}$.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.