Math 1553 Worksheet §2.5, 2.6, 2.7, 2.9, 3.1

1. If the statement is always true, circle TRUE. Otherwise, circle FALSE. Justify your answer.

a) Suppose $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ and $A\begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Must v_1, v_2, v_3 be linearly dependent? If true, write a linear dependence relation for the vectors.
TRUE **FALSE**

b) If b is in Col(A), then so is $5b$. **TRUE** **FALSE**

c) In the following, A is an $m \times n$ matrix.

(1) **TRUE** **FALSE** If A has linearly dependent columns, then $m < n$.

(2) **TRUE** **FALSE** If A has linearly independent columns, then $Ax = b$ must have at least one solution for each b in $\mathbf{R}^m$.

(3) **TRUE** **FALSE** If b is a vector in $\mathbf{R}^m$ and $Ax = b$ has exactly one solution, then $m \geq n$.

2. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

a) If A is a 3×10 matrix with 2 pivots, then $\dim(\mathrm{Nul}\,A) = 8$ and $\mathrm{rank}(A) = 2$.

TRUE **FALSE**

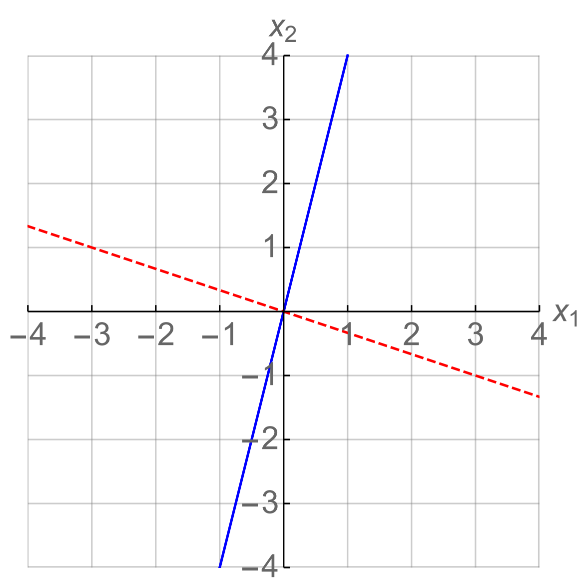
b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the transformation $T(x) = Ax$ must have $\mathbf{R}^m$ as its range.

TRUE **FALSE**

c) If $\{a, b, c\}$ is a basis of a subspace V, then $\{a, a+b, b+c\}$ is a basis of V as well.

TRUE **FALSE**

3. Write a matrix A so that Col(A) is the solid blue line and Nul(A) is the dotted red line drawn below.



4. Let $A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$, and let T be the matrix transformation associated to A, so $T(x) = Ax$.

a) What is the domain of T? What is the codomain of T? Give an example of a vector in the range of T.

b) This is extra practice in case the studio finishes the rest of the worksheet early.

The RREF of A is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

(i) Write bases for Col(A) and Nul(A).

(ii) Is there a vector in the codomain of T which is not in the range of T? Justify your answer.