Practice Midterm 1

This is a preview of the draft version of the quiz

Started: Sep 15 at 8:14pm

Quiz Instructions

Once you open this quiz, you will have 75 minutes to submit it. You will have only one submission attempt. The quiz must be submitted by 7:59 PM (Atlanta time) on Thursday, September 17. There are 20 questions after the honor code pledge.

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### Question 1

Please read and attest to the honor statement below:

I understand that this assessment is open-book and open-note, but not open-internet. I may use my class notes, my instructor's notes, and the ILA textbook at [https://textbooks.math.gatech.edu/ila/ila.pdf](https://textbooks.math.gatech.edu/ila/ila.pdf).

However, I will not visit any other websites, use any search engines, or use any calculators or computer aids whatsoever (Matlab, Mathematica, Chegg.com, Geogebra, etc.) as I take this assessment.

This assessment is completely my own work. I will not discuss the answers or any of the contents of this assessment with anyone until the time it is due.

- I attest to my integrity, and I understand that any suspected violation of this policy may be prosecuted to the fullest extent allowable by Georgia Tech.

### Question 2

Suppose that the augmented matrix for the matrix equation $Ax = b$ has reduced row echelon form.
Fill in the parametric vector form of the solution set:

\[
\begin{pmatrix}
A \\
B
\end{pmatrix}
+ x_2
\begin{pmatrix}
C \\
D
\end{pmatrix}
\]

A= , B= , C= , D=

Question 3

Suppose that \( A \) is a \( 3 \times 3 \) matrix, that

\[
A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad \text{and} \quad A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}
\]

and that the set of solutions to the matrix equation \( Ax = 0 \) is a line in \( \mathbb{R}^3 \). What best describes the set of solutions to the following matrix equation?

\[
Ax = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}
\]

- A line in \( \mathbb{R}^3 \) through \((1,1,0)\)
- A line in \( \mathbb{R}^3 \) through \((0,0,0)\)
- A line in \( \mathbb{R}^3 \) through \((2,0,0)\)
- A line in \( \mathbb{R}^3 \) through \((1,-1,0)\)
- None of the above
Question 4

Find the value of $h$ for which the following set of vectors is linearly dependent.
\[
\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ h \end{pmatrix} \right\}
\]

$h =$

Question 5

Suppose that $u$, $v$, and $w$ are vectors in $\mathbb{R}^3$. Suppose that any two of the vectors spans a plane.

Must it be true that the vectors $u$, $v$, and $w$ are linearly independent?

- Yes
- No

Question 6

Compute the entries of the following product.
\[
\begin{pmatrix} 1 & 2 & 0 \\ 0 & -2 & 3 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}
\]

$a =$

$b =$

$c =$
Question 7  
1 pts

Consider the following system of linear equations.

\[ x - 2y = 4 \]
\[ 3y + 5z = -1 \]

Which of the following matrix equations corresponds to the given linear system?

- \[
\begin{pmatrix}
1 & -2 & 0 \\
0 & 3 & -5
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
4 \\
-1
\end{pmatrix}
\]

- \[
\begin{pmatrix}
1 & -2 \\
3 & -5
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
4 \\
-1
\end{pmatrix}
\]

- \[
\begin{pmatrix}
1 & -2 & 0 \\
0 & 3 & -5
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
4 \\
-1 \\
0
\end{pmatrix}
\]

- \[
\begin{pmatrix}
1 & -2 & 0 \\
0 & 3 & -5
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
4 \\
-1
\end{pmatrix}
\]

Question 8  
1 pts

Is the following matrix equation consistent?

\[
\begin{pmatrix}
1 & 0 & -2 \\
0 & 3 & -1 \\
2 & 3 & -5
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} =
\begin{pmatrix}
1 \\
3 \\
5
\end{pmatrix}
\]
Question 9

Suppose $A$ is a $4 \times 6$ matrix. Then the dimension of the solution set of the matrix equation $Ax = 0$ is at most 5.

- True
- False

Question 10

Which of the following statement are true? Select all that apply.

- If we have a linearly dependent set of vectors, then every vector is a linear combination of the others.
- If a matrix $A$ has linearly independent columns, and $Ax=b$ is consistent for all choices of $b$ then $A$ is a square matrix.
- The zero vector is in the span of every set of vectors.
- Linear algebra is completely useless.

Question 11
Consider the traffic pattern on the following collection of streets. Each on the 9 streets is labeled by the number of cars that pass over that street per hour.

Construct a linear system to determine the number of cars per hour that pass through the three streets in the middle. How many free variables are there in the solution? (For fun: why does the answer make sense?)

Question 12

Is it possible to find four different vectors \(v_1, v_2, v_3, v_4\) in \(\mathbb{R}^4\) so that any two of the vectors span a plane and all four of them span \(\mathbb{R}^4\).

- [ ] Yes
- [ ] No
Consider the following matrix.

\[ A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} \]

Which of the followings is a correct statement about \( A \)?

- \( Ax = 0 \) has multiple solutions
- There exists a vector \( b \) in \( \mathbb{R}^3 \) so that \( Ax = b \) has no solution
- \( Ax = b \) is consistent for all \( b \) in \( \mathbb{R}^3 \).
- \( A \) has fewer than 3 pivots

Which of the following matrices have the property that their columns span \( \mathbb{R}^3 \)? Select all that apply.

- \( \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 5 \end{pmatrix} \)
- \( \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 3 & 5 \\ 3 & 5 & 6 & 9 \end{pmatrix} \)
- \( \begin{pmatrix} 1 & 0 & 0 \\ 18 & 2 & 0 \\ 20 & 1 & 12 \end{pmatrix} \)
**Question 15**  
1 pts

Which of the following matrices are in reduced row echelon form? Select all that apply.

- \[
\begin{pmatrix}
0 & 0 & 1 & -3 \\
0 & 1 & 0 & 2 \\
1 & 0 & 0 & 4
\end{pmatrix}
\]

- \[
\begin{pmatrix}
0 & 1 & 0 & 1 & -2 & 0 \\
0 & 0 & 0 & 2 & -5 & 0 \\
0 & 0 & 0 & 0 & 7 & 0
\end{pmatrix}
\]

- \[
\begin{pmatrix}
0 & 1 & 0 & 0 & -2 & 0 \\
0 & 0 & 0 & 1 & -5 & 0 \\
0 & 0 & 0 & 0 & 0 & 2
\end{pmatrix}
\]

- \[
\begin{pmatrix}
1 \\
0
\end{pmatrix}
\]

**Question 16**  
1 pts

Which of the following correctly says what is means for a point in \( \mathbb{R}^n \) to be a solution to a system of equations in \( n \) variables?

- The point satisfies all of the equations in the system.
- The point satisfies at least one equation in the system.

**Question 17**  
1 pts
Any set of 7 vectors in \( \mathbb{R}^5 \) is linearly dependent.

- True
- False

**Question 18**

Consider the set of vectors \( \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -\pi \\ \pi \end{pmatrix} \right\} \). Is this set linearly independent?

- Yes
- No

**Question 19**

Suppose that \( A \) is a 3 \( \times \) 4 matrix with exactly 2 pivots. What best describes the set of solutions to the matrix equation \( Ax = 0 \)?

- a plane in \( \mathbb{R}^4 \)
- a line in \( \mathbb{R}^4 \)
- a plane in \( \mathbb{R}^3 \)
- a line in \( \mathbb{R}^3 \)

**Question 20**
For each system of equations, indicate how many free variables there are in the set of solutions. You should assume that the number of variables is the largest subscript in the system of equations.

(a) \[ \text{[ Select]} \]
\[
\begin{align*}
  x_1 &\quad -2x_2 &= 4 \\
  x_3 &= 0
\end{align*}
\]

(b) \[ \text{[ Select]} \]
\[
\begin{align*}
  x_1 &= -1 \\
  x_2 &= 0 \\
  x_3 &= -1
\end{align*}
\]

**Question 21**

Suppose that we have a system of 2 equations in 4 variables. Which of the following could be the set of solutions? Select all that apply.

- \( x_1 = -3x_2 - 2 \)
- \( x_2 = x_2 \)
- \( x_3 = 8 \)
- \( x_4 = -1 \)
- \( x_1 = -1 \)
- \( x_2 = -3x_4 + 9 \)
- \( x_3 = x_3 \)
- \( x_4 = x_4 \)
\[
\begin{align*}
  x_1 &= -2 \\
  x_2 &= 5 \\
  x_3 &= -7 \\
  x_4 &= 1
\end{align*}
\]

\[
\begin{align*}
  x_1 &= -9x_2 + 3x_3 - 2x_4 + 8 \\
  x_2 &= x_2 \\
  x_3 &= x_3 \\
  x_4 &= x_4
\end{align*}
\]

☐ No solutions