1. Suppose $p$ and $q$ are real numbers on the open interval $(0, 1)$, and

$$A = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

(1) Is $A$ stochastic? Why?
(2) Does $A$ have unique steady state vector? Why?
(3) By inspection (without computation), give an eigenvalue of $A$.
(4) Compute the steady-state vector of $A$.
(5) Compute the limit

$$\lim_{n \to \infty} A^n$$

2. $y = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$, $u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $(u)_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(1) Determine whether $u_1$ and $u_2$
   (a) are linearly independent
   (b) are mutually orthogonal
   (c) are orthonormal
   (d) span $\mathbb{R}^3$
(2) Is $y$ in $W = \text{Span}\{u_1, u_2\}$?
(3) Compute the vector, $\tilde{y} \in W$, that most closely approximates $y$. [You may need orthogonal projections from §6.2]
(4) Construct a vector, $z$, that is in $W^\perp$.
(5) Make a rough sketch (use online tools) of $u_1, u_2, y, \tilde{y},$ and $z$. 