NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Twisted rabbits and Hubbard trees
Becca Winarski
University of Wisconsin-Milwaukee

joint with Jim Belk, Justin Lanier and Dan Margalit
The twisted rabbit problem

\[ p(z) = z^2 + c \]
The twisted rabbit problem

\[ p(z) = z^2 + c \]
The twisted rabbit problem

\[ p(z) = z^2 + c \]
The twisted rabbit problem

\[ p(z) = z^2 + c \]

3 values of \( c \):

\[ p_R, p_C, p_A \]
Julia sets

Rabbit
$\text{Im}(c) > 0$

Corabbit
$\text{Im}(c) < 0$

Airplane
$\text{Im}(c) = 0$

Images courtesy of Bill Floyd https://www.math.vt.edu/netmaps/index.php
The twisted rabbit problem
The twisted rabbit problem

$T_d \circ p_R$
The twisted rabbit problem

Thurston

\[ T_d \circ p_R \]
The twisted rabbit problem

Twisted rabbit problem:
\[ f \in \text{Mod}(\mathbb{C}, P) \text{ what is } f \circ p_R? \]
Solving TRP

1. Topological description of $\rho_R$
Solving TRP

1. Topological description of $\rho_R$ \rightarrow branched covers
Solving TRP

1. Topological description of $p_R$ \rightarrow branched covers

2. Distinguish $p_R, p_C, p_A$
Solving TRP

1. Topological description of $p_R$ \xrightarrow{} branched covers

2. Distinguish $p_R, p_C, p_A$ \xrightarrow{} Hubbard trees
Solving TRP

1. Topological description of $p_R$ \rightarrow branched covers

2. Distinguish $p_R, p_C, p_A$ \rightarrow Hubbard trees

3. Given $f$, what is $f \circ p_R$?
Solving TRP

1. Topological description of $p_R$ \rightarrow branched covers

2. Distinguish $p_R, p_C, p_A$ \rightarrow Hubbard trees

3. Given $f$, what is $f \circ p_R$?
   \rightarrow following Bartholdi—Nekyrashevych
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:
• edges are contained in Julia set
• leaves are in P
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:
- edges are contained in Julia set
- leaves are in \( P \)
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:
- edges are contained in Julia set
- leaves are in $\mathbb{P}$

$$p_R$$

$$(c^2 + c, c, 0)$$
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:
• edges are contained in Julia set
• leaves are in P
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:

- edges are contained in Julia set
- leaves are in $P$
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:
• edges are contained in Julia set
• leaves are in P
Hubbard Trees

Each polynomial has a unique tree called the Hubbard tree:

• edges are contained in Julia set
• leaves are in P

$p_R \quad p_C \quad p_A

T_R \quad T_C \quad T_A
Hubbard trees as an invariant

$T_A$ is combinatorially different from $T_R$ and $T_C$
Hubbard trees as an invariant

$T_A$ is combinatorially different from $T_R$ and $T_C$

AND:

• $p_R^{-1}$ rotates the edges of $T_R$ clockwise
• $p_C^{-1}$ rotates the edges of $T_C$ counterclockwise
Hubbard trees as an invariant

$T_A$ is combinatorially different from $T_R$ and $T_C$

AND:

- $p^{-1}_R$ rotates the edges of $T_R$ clockwise
- $p^{-1}_C$ rotates the edges of $T_C$ counterclockwise

**Proposition** (Belk, Lanier, Margalit, W)
The Hubbard tree and its direction of rotation under $p^{-1}$ distinguish $p_R, p_C, p_A$. 
The general conjectures

**Conjecture 1:** Given a polynomial \( p \) and a tree \( T \), \( \{p^{-n}(T)\} \) will converge to the Hubbard tree for \( p \).
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
Tree convergence

Images by Jim Belk
The general conjectures

**Conjecture 1:** Given a polynomial $p$ and a tree $T$, \( \{ p^{-n}(T) \} \) will converge to the Hubbard tree for $p$.

**Conjecture 2:** Given polynomials $p_1, p_2$, the Hubbard trees and direction of rotation under $p_1^{-1}, p_2^{-1}$ are different.
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Homological eigenvalues of pseudo Anosov mapping classes

Asaf Hadari

University of Hawaii at Manoa

October 16, 2017
Let $\Sigma$ be a hyperbolic orientable surface of finite type and let $\text{Mod}(\Sigma)$ be its mapping class group.
Let $\Sigma$ be a hyperbolic orientable surface of finite type and let $\text{Mod}(\Sigma)$ be its mapping class group.

Suppose $\Sigma$ has at least one puncture, or marked point. Given a finite cover $\pi : \tilde{\Sigma} \to \Sigma$, a finite index subgroup $\Gamma < \text{Mod}(\Sigma)$ lifts to $\text{Mod}(\tilde{\Sigma})$. The action of $\Gamma$ on $H_1(\tilde{\Sigma}, \mathbb{Z})$ is called the \textit{homological representation corresponding to} $\pi$. Denote this representation $\rho_{\pi}$.
Let $\Sigma$ be a hyperbolic orientable surface of finite type and let $\text{Mod}(\Sigma)$ be its mapping class group.

Suppose $\Sigma$ has at least one puncture, or marked point. Given a finite cover $\pi : \tilde{\Sigma} \to \Sigma$, a finite index subgroup $\Gamma < \text{Mod}(\Sigma)$ lifts to $\text{Mod}(\tilde{\Sigma})$. The action of $\Gamma$ on $H_1(\tilde{\Sigma}, \mathbb{Z})$ is called the \textit{homological representation corresponding to } $\pi$. Denote this representation $\rho_\pi$.

**Question:** How much information about $\text{Mod}(\Sigma)$ can be recovered from its homological representations?
Given a non-identity element $f \in \text{Mod}(\Sigma)$, it is easy to show that there is a cover $\pi$ to which $f$ lifts such that $\rho_{\pi}(f) \neq \text{Id}$. Suppose $f$ is a pseudo Anosov mapping class. Can we recover more information about $f$?
Given a non-identity element $f \in \text{Mod}(\Sigma)$, it is easy to show that there is a cover $\pi$ to which $f$ lifts such that $\rho_{\pi}(f) \neq \text{Id}$. Suppose $f$ is a pseudo Anosov mapping class. Can we recover more information about $f$?

Let $\sigma_{\pi}(f)$ be the spectral radius of the operator $\rho_{\pi}(f)$. If $f$ has orientable stable and unstable foliations then $\sigma_{\pi}(f)$ is $\lambda(f)$, the dilatation of $f$. It is simple to show that $\sigma_{\pi}(f) \leq \lambda(f)$.
Given a non-identity element \( f \in \text{Mod}(\Sigma) \), it is easy to show that there is a cover \( \pi \) to which \( f \) lifts such that \( \rho_\pi(f) \neq \text{Id} \). Suppose \( f \) is a pseudo Anosov mapping class. Can we recover more information about \( f \)?

- Let \( \sigma_\pi(f) \) be the spectral radius of the operator \( \rho_\pi(f) \). If \( f \) has orientable stable and unstable foliations then \( \sigma_\pi(f) \) is \( \lambda(f) \), the dilatation of \( f \). It is simple to show that \( \sigma_\pi(f) \leq \lambda(f) \).
- McMullen proved that \( \sup_\pi \sigma_\pi(f) \) can be smaller than \( \lambda(f) \).
Given a non-identity element $f \in \text{Mod}(\Sigma)$, it is easy to show that there is a cover $\pi$ to which $f$ lifts such that $\rho_\pi(f) \neq \text{Id}$. Suppose $f$ is a pseudo Anosov mapping class. Can we recover more information about $f$?

- Let $\sigma_\pi(f)$ be the spectral radius of the operator $\rho_\pi(f)$. If $f$ has orientable stable and unstable foliations then $\sigma_\pi(f)$ is $\lambda(f)$, the dilatation of $f$. It is simple to show that $\sigma_\pi(f) \leq \lambda(f)$.

- McMullen proved that $\sup_\pi \sigma_\pi(f)$ can be smaller than $\lambda(f)$.

- **Conjecture (McMullen):** For $f$ pseudo-Anosov $\sup_\pi \sigma_\pi(f) > 1$. 

Images of individual elements of $\text{Mod}(\Sigma)$
Recently, I proved the following result which provided evidence for the McMullen conjecture.

**Theorem**
Let $\mathcal{M}$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\mathcal{M})$ be an infinite order mapping class. Then there exists a cover $\varphi$ such that $\varphi(f)$ has infinite order.

Building on this proof, I've proved the following.

**Theorem**
Let $\mathcal{M}$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\mathcal{M})$ be a pseudo Anosov mapping class. Then there exists a finite cover $\varphi$ such that $\varphi(f) > 1$. 

Asaf Hadari  
Homological eigenvalues of pseudo Anosov mapping classes
Recently, I proved the following result which provided evidence for the McMullen conjecture.

**Theorem**

Let $\Sigma$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\Sigma)$ be an infinite order mapping class. Then there exists a cover $\pi$ such that $\rho_{\pi}(f)$ has infinite order.
Recently, I proved the following result which provided evidence for the McMullen conjecture.

**Theorem**

Let $\Sigma$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\Sigma)$ be an infinite order mapping class. Then there exists a cover $\pi$ such that $\rho_{\pi}(f)$ has infinite order.

Building on this proof, I’ve proved the following.
Recently, I proved the following result which provided evidence for the McMullen conjecture.

**Theorem**

Let $\Sigma$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\Sigma)$ be an infinite order mapping class. Then there exists a cover $\pi$ such that $\rho_\pi(f)$ has infinite order.

Building on this proof, I’ve proved the following.

**Theorem**

Let $\Sigma$ be a surface with free non-abelian fundamental group. Let $f \in \text{Mod}(\Sigma)$ be a pseudo Anosov mapping class. Then there exists a finite cover $\pi$ such that $\sigma_\pi(f) > 1$. 

Asaf Hadari

Homological eigenvalues of pseudo Anosov mapping classes
Some features of the proof

- The cover $\pi$ can be taken to be characteristic, and the deck group can be taken to be solvable.
Some features of the proof

- The cover $\pi$ can be taken to be characteristic, and the deck group can be taken to be solvable.
- The proof can be made to be constructive.
Some features of the proof

- The cover $\pi$ can be taken to be characteristic, and the deck group can be taken to be solvable.
- The proof can be made to be constructive.
- The proof also works if we replace $\text{Mod}(\Sigma)$ with $\text{Aut}(F_n)$, and $f$ with a fully irreducible element of $\text{Aut}(F_n)$.
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Mapping class groups and monodromy of families of plane curves

Nick Salter
Harvard University
October 30, 2017
Plane curves
Plane curves

Object of study:
Object of study: Smooth projective complex plane curves of degree $d$
Object of study: Smooth projective complex plane curves of degree $d$

E.g. $\{[X : Y : Z] \in \mathbb{C}P^2 \mid X^d + Y^d + Z^d = 0\}$
Object of study: Smooth projective complex plane curves of degree $d$

E.g. $\{[X : Y : Z] \in \mathbb{C}P^2 \mid X^d + Y^d + Z^d = 0\}$

Universal curve: Surface bundle $\mathcal{X}_d \to \mathcal{P}_d$
Object of study: Smooth projective complex plane curves of degree $d$

E.g. $\{[X : Y : Z] \in \mathbb{C}P^2 \mid X^d + Y^d + Z^d = 0\}$

Universal curve: Surface bundle $\mathcal{X}_d \to \mathcal{P}_d$

Monodromy: Homomorphism $\rho_d : \pi_1(\mathcal{P}_d) \to \text{Mod}(\Sigma_g)$
Object of study: Smooth projective complex plane curves of degree $d$

E.g. $\{[X : Y : Z] \in \mathbb{C}P^2 \mid X^d + Y^d + Z^d = 0\}$

Universal curve: Surface bundle $\mathcal{X}_d \to \mathcal{P}_d$

Monodromy: Homomorphism $\rho_d : \pi_1(\mathcal{P}_d) \to \text{Mod}(\Sigma_g)$

Basic question: What is $\Gamma_d := \text{im}(\rho_d)$?
What is known?
What is known?

Beauville:
What is known?

Beauville: Computation of “cohomological monodromy”
What is known?

Beauville: Computation of “cohomological monodromy”

Key points:
What is known?

Beauville: Computation of “cohomological monodromy”

Key points: Always finite-index in $\text{Sp}(2g, \mathbb{Z})$
What is known?

Beauville: Computation of “cohomological monodromy”

Key points: Always finite-index in $\text{Sp}(2g, \mathbb{Z})$

For $d$ even, surjective
What is known?

Beauville: Computation of “cohomological monodromy”

Key points: Always finite-index in $\text{Sp}(2g, \mathbb{Z})$

For $d$ even, *surjective*

For $d$ odd, must preserve a *spin structure*
What is known?

Beauville: Computation of “cohomological monodromy”

Key points: Always finite-index in $\text{Sp}(2g, \mathbb{Z})$

- For $d$ even, surjective
- For $d$ odd, must preserve a spin structure

Question: Are there “non-cohomological” obstructions?

Does $\Gamma_d = \text{Mod}(\Sigma_g)$ for $d$ even?
Some prior work
Some prior work

Folklore observation:
Folklore observation: There are “higher spin structures” preserved by $\Gamma_d$ for any $d$. 
Folklore observation: There are “higher spin structures” preserved by $\Gamma_d$ for any $d$.

Define $\text{Mod}(\Sigma_g)[\phi_d]$ as the stabilizer of $\phi_d$ (a higher spin structure)
Folklore observation: There are “higher spin structures” preserved by $\Gamma_d$ for any $d$.

Define $\text{Mod}(\Sigma_g)[\phi_d]$ as the stabilizer of $\phi_d$ (a higher spin structure).

Theorem (S., ’16):
Some prior work

Folklore observation: There are “higher spin structures” preserved by $\Gamma_d$ for any $d$

Define $\text{Mod}(\Sigma_g)[\phi_d]$ as the stabilizer of $\phi_d$ (a higher spin structure)

Theorem (S., ’16): $\Gamma_5 = \text{Mod}(\Sigma_6)[\phi_5]$
Folklore observation: There are “higher spin structures” preserved by $\Gamma_d$ for any $d$.

Define $\text{Mod}(\Sigma_g)[\phi_d]$ as the stabilizer of $\phi_d$ (a higher spin structure).

Theorem (S., ’16): $\Gamma_5 = \text{Mod}(\Sigma_6)[\phi_5]$.

Method: reduce to Johnson’s work on Torelli group.
New theorem
New theorem

Theorem (S., Tuesday):
New theorem

Theorem (S., Tuesday): \[ \Gamma_d = \text{Mod}(\Sigma_g)[\phi_d] \] for all d even
Theorem (S., Tuesday): \( \Gamma_d = \text{Mod}(\Sigma_g)[\phi_d] \) for all \( d \) even

\[ \Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d] \quad \text{finite index} \quad \text{for all} \ d \ \text{odd} \]
New theorem

Theorem (S., Tuesday): \( \Gamma_d = \text{Mod}(\Sigma_g)[\phi_d] \) for all \( d \) even

\( \Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d] \) finite index for all \( d \) odd

Points of interest:
Theorem (S., Tuesday): $\Gamma_d = \text{Mod}(\Sigma_g)[\phi_d]$ for all $d$ even

$\Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d]$ finite index for all $d$ odd

Points of interest:

Determine generators for $\text{Mod}(\Sigma_g)[\phi_d]$
Theorem (S., Tuesday): $\Gamma_d = \text{Mod}(\Sigma_g)[\phi_d]$ for all $d$ even

$\Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d]$ finite index for all $d$ odd

Points of interest:

Determine generators for $\text{Mod}(\Sigma_g)[\phi_d]$

Tip of an iceberg: line bundles on toric varieties
Theorem (S., Tuesday): \( \Gamma_d = \text{Mod}(\Sigma_g)[\phi_d] \) for all \( d \) even

\[ \Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d] \text{ finite index for all } d \text{ odd} \]

Points of interest:

Determine generators for \( \text{Mod}(\Sigma_g)[\phi_d] \)

Tip of an iceberg: line bundles on toric varieties

(results also for curves in \( \mathbb{C}P^1 \times \mathbb{C}P^1 \), Hirzebruch surfaces, etc.)
New theorem

Theorem (S., Tuesday): \[ \Gamma_d = \Mod(\Sigma_g)[\phi_d] \text{ for all } d \text{ even} \]

\[ \Gamma_d \leq \Mod(\Sigma_g)[\phi_d] \text{ finite index for all } d \text{ odd} \]

Points of interest:

Determine generators for \( \Mod(\Sigma_g)[\phi_d] \)

Tip of an iceberg: line bundles on toric varieties

(results also for curves in \( \mathbb{CP}^1 \times \mathbb{CP}^1 \), Hirzebruch surfaces, etc.)

Uses tropical geometry methods developed by Crétois-Lang
New theorem

Theorem (S., Tuesday): \[ \Gamma_d = \text{Mod}(\Sigma_g)[\phi_d] \] for all \( d \) even

\[ \Gamma_d \leq \text{Mod}(\Sigma_g)[\phi_d] \text{ finite index} \] for all \( d \) odd

Points of interest:

Determine generators for \( \text{Mod}(\Sigma_g)[\phi_d] \)

Tip of an iceberg: line bundles on toric varieties

(results also for curves in \( \mathbb{C}P^1 \times \mathbb{C}P^1 \), Hirzebruch surfaces, etc.)

Uses tropical geometry methods developed by Crétois-Lang

Answers a question of Donaldson from 2000
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Semidualities from products of trees

Daniel Studenmund
joint with Kevin Wortman

University of Notre Dame

No Boundaries, October 2017
Theorem (Borel–Serre)

Suppose $G$ is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-duality group: there is a number $d$ such that for any $\mathbb{Q}\Gamma$-module $M$ there are isomorphisms

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes_{\mathbb{Q}} M),$$

where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$. 
Theorem (Borel–Serre)

Suppose $G$ is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-duality group: there is a number $d$ such that for any $\mathbb{Q}\Gamma$-module $M$ there are isomorphisms

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes \mathbb{Q} M),$$

where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$.

If $\Gamma$ is cocompact in $G(\mathbb{R})$, then $D = \mathbb{Q}$.
Theorem (Borel–Serre)

Suppose $G$ is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-duality group: there is a number $d$ such that for any $\mathbb{Q}\Gamma$-module $M$ there are isomorphisms

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes \mathbb{Q} M),$$

where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$.

If $\Gamma$ is cocompact in $G(\mathbb{R})$, then $D = \mathbb{Q}$.

If $\Gamma$ not cocompact, then $D$ is countably generated and $G(\mathbb{Q}) \bowtie D$. 
**Theorem (Borel–Serre)**

Suppose $G$ is a semisimple algebraic group defined over $\mathbb{Q}$ and $\Gamma \leq G$ an arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-duality group: there is a number $d$ such that for any $\mathbb{Q}\Gamma$-module $M$ there are isomorphisms

$$H^k(\Gamma, M) \cong H_{d-k}(\Gamma, D \otimes \mathbb{Q} M),$$

where $D = H^d(\Gamma, \mathbb{Q}\Gamma)$.

If $\Gamma$ is cocompact in $G(\mathbb{R})$, then $D = \mathbb{Q}$.

If $\Gamma$ not cocompact, then $D$ is countably generated and $G(\mathbb{Q}) \sim D$.

**Example:** If $\Gamma = SL_2(\mathbb{Z})$ then $d = 1$ and $D = H^1(\Gamma, \mathbb{Q}\Gamma) \cong \bigoplus_{P^1(\mathbb{Q})} \mathbb{Q}$.
\( \Gamma = \text{SL}_2(\mathbb{Z}) \) acts on hyperbolic plane \( \mathbb{H}^2 \):

https://golem.ph.utexas.edu/category/2008/02/modular_forms.html
\( \Gamma = \text{SL}_2(\mathbb{Z}) \) acts on hyperbolic plane \( \mathbb{H}^2 \):

\[ \mathbb{H}^2 \cap \mathbb{R} = \mathbb{R} \subset \mathbb{H}^2 \]

\[ \mathbb{H}^2 = \mathbb{H}^2 / \mathbb{R} \]

\( \Gamma \) acts cocompactly on \( \mathbb{H}^2 = \mathbb{H}^2 - \bigcup_{x \in \mathbb{P}^1(\mathbb{Q})} B_x \), for horoballs \( B_x \)
Tit = \text{SL}_2(\mathbb{Z}) \text{ acts on hyperbolic plane } \mathbb{H}^2:\n
\Gamma \text{ acts cocompactly on } \widehat{\mathbb{H}}^2 = \mathbb{H}^2 - \bigcup_{x \in P^1(\mathbb{Q})} B_x, \text{ for horoballs } B_x \n
H^1(\Gamma, \mathbb{Q} \Gamma) \cong H^1_c(\widehat{\mathbb{H}}^2) \cong H_0(\widehat{\mathbb{H}}^2, \partial \widehat{\mathbb{H}}^2) \cong \tilde{H}_0(\partial \widehat{\mathbb{H}}^2) \cong \tilde{H}_0(P^1(\mathbb{Q}))
**Fact:** $\Gamma$ is a $\mathbb{Q}$-duality group of dimension $d$ iff

- $H^n(\Gamma, \mathbb{Q}\Gamma) = 0$ if $n \neq d$, and
- $\Gamma$ is type $FP$ over $\mathbb{Q}$.
Fact: \( \Gamma \) is a \( \mathbb{Q} \)-duality group of dimension \( d \) iff
- \( H^n(\Gamma, \mathbb{Q}\Gamma) = 0 \) if \( n \neq d \), and
- \( \Gamma \) is type \( FP \) over \( \mathbb{Q} \).

Definition: \( \Gamma \) is a \( \mathbb{Q} \)-semiduality group of dimension \( d \) if
- \( H^n(\Gamma, \mathbb{Q}\Gamma) = 0 \) if \( n \neq d \),
- \( \Gamma \) is type \( FP_{d-1} \) over \( \mathbb{Q} \), and
- \( cd_{\mathbb{Q}}(\Gamma) = d \).
Fact: $\Gamma$ is a $\mathbb{Q}$-duality group of dimension $d$ iff
- $H^n(\Gamma, \mathbb{Q}\Gamma) = 0$ if $n \neq d$, and
- $\Gamma$ is type $FP$ over $\mathbb{Q}$.

Definition: $\Gamma$ is a $\mathbb{Q}$-semiduality group of dimension $d$ if
- $H^n(\Gamma, \mathbb{Q}\Gamma) = 0$ if $n \neq d$,
- $\Gamma$ is type $FP_{d-1}$ over $\mathbb{Q}$, and
- $cd_{\mathbb{Q}}(\Gamma) = d$.

Theorem: If $\Gamma$ is a semiduality group then for any $M$ there are maps

$$\phi : H_{d-k}(\Gamma, D \otimes_{\mathbb{Q}} M) \to H^k(\Gamma, M)$$

that are isomorphisms for sufficiently ‘nice’ $M$. 
Conjecture: Suppose $G$ is a simple algebraic group defined over a global function field $K$ of characteristic $p$ and $\Gamma$ is an $S$-arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-semiduality group.
**Conjecture:** Suppose $G$ is a simple algebraic group defined over a global function field $K$ of characteristic $p$ and $\Gamma$ is an $S$-arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-semiduality group.

**Theorem (S.–Wortman)**

*Conjecture holds if $G = SL_2$, in which case $SL_2(K) \heartsuit H^d(\Gamma, \mathbb{Q}\Gamma)$.*
**Conjecture:** Suppose $G$ is a simple algebraic group defined over a global function field $K$ of characteristic $p$ and $\Gamma$ is an $S$-arithmetic subgroup. Then $\Gamma$ is a $\mathbb{Q}$-semiduality group.

**Theorem (S.–Wortman)**

*Conjecture holds if $G = SL_2$, in which case $SL_2(K) \bowtie H^d(\Gamma, \mathbb{Q}\Gamma)$.*

**Example:** $K = \mathbb{F}_2(t)$ and $\Gamma = SL_2(\mathbb{F}_2[t, t^{-1}])$
Compute $H^n(\Gamma, \mathbb{Q}\Gamma)$ for $\Gamma = \text{SL}_2(\mathbb{F}_2[t, t^{-1}]) \ltimes T \times T$
Compute $H^n(\Gamma, \mathbb{Q}\Gamma)$ for $\Gamma = \text{SL}_2(\mathbb{F}_2[t, t^{-1}]) \curvearrowright T \times T$

Horoball $B =$ filtered by $B_n =$
Compute $H^n(\Gamma, \mathbb{Q}\Gamma)$ for $\Gamma = \text{SL}_2(\mathbb{F}_2[t, t^{-1}]) \looparrowright T \times T$

Horoball $B = \text{filtered by } B_n = \text{Dualizing module satisfies:}$

$$0 \rightarrow H^2_c(T \times T) \rightarrow H^2(\Gamma, \mathbb{Q}\Gamma) \rightarrow \bigoplus_{x \in P^1(K)} \text{lim}^1 H^1_c(B_n) \rightarrow 0$$

Thank you! Happy birthday, Benson!
Compute $H^n(\Gamma, \mathbb{Q}\Gamma)$ for $\Gamma = \text{SL}_2(\mathbb{F}_2[t, t^{-1}]) \ltimes T \times T$

Horoball $B = \text{filtered by } B_n = \text{Dualizing module satisfies:}$

$$0 \to H^2_c(T \times T) \to H^2(\Gamma, \mathbb{Q}\Gamma) \to \bigoplus_{x \in P^1(K)} \lim_{n \to \infty} H^1_c(B_n) \to 0$$

**Thank you! Happy birthday, Benson!**
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Fast Nielsen-Thurston Classification

Balázs Strenner
Georgia Institute of Technology
joint with Dan Margalit and S. Öykü Yurttaş

No boundaries – Groups in algebra, geometry and topology
University of Chicago
October 28, 2017
Benson
The Nielsen-Thurston Classification

\[ \text{Mod}(S) \]

Finite order

Pseudo-Anosov

Reducible
The Nielsen-Thurston Classification Problem

Fix finite generating set for Mod(S).

INPUT

$f = s_1s_2...s_n$ → Algorithm

Running time: function of $|f|$.

OUTPUT

Finite order
Reducible reducing curves
Pseudo-Anosov stretch factor foliations
Main Theorem

**Theorem (Margalit-S-Yurttaş):** There exists a quadratic-time algorithm for the Nielsen-Thurston Classification Problem.
Isometries of $\mathbb{H}^2$

- **Elliptic**: (up to power)
- **Hyperbolic**: source
- **Parabolic**: source/sink

Diagram:
- Blue circle: Elliptic
- Black circle with arrows: Hyperbolic
- Black circle with arrows in opposite direction: Parabolic
The mapping class group

PMF(S)  Teich(S)  sink ($F^u$)  source ($F^s$)  source/sink (invariant curves)

finite order  pseudo-Anosov  Reducible
(up to power)
Thurston, Mosher: Compute the piecewise linear map and find all the eigenvectors.

Teich(S)

source (F_s)

sink (F_u)

PMF(S)

source/sink (invariant curves)

finite order
(up to power)

pseudo-Anosov

Reducible
Thurston, Mosher: Compute the piecewise linear map and find all the eigenvectors.

Exponentially many pieces

Thurston, Mosher: Compute the piecewise linear map and find all the eigenvectors.

Exponentially many pieces

finite order (up to power)
pseudo-Anosov
Reducible
Iterate!

*Toby Hall (Dynn)*: First iterate, then compute eigenvectors.
Iterate!

*Toby Hall (Dyyn)*: First iterate, then compute eigenvectors.

- Unknown rate of convergence
- Unknown behavior in reducible case
Iterate!

*Toby Hall (Dynn):* First iterate, then compute eigenvectors.

Unknown rate of convergence
Unknown behavior in reducible case

*Bell-Schleimer:* convergence can be exponentially slow

Diagram:
- Source to sink
- Iteration arrows
- Emojis expressing frustration

Note:
- The diagram illustrates the concept of iteration with arrows pointing from source to sink.
Iterate!

_Toby Hall (Dyynn):_ First iterate, then compute eigenvectors.

Unknown rate of convergence
Unknown behavior in reducible case

_Bell-Schleimer:_ convergence can be exponentially slow

_Margalit-S-Yurttaş:_ $O(1)$ iterations is enough.
Macaw (implementation)

1. *Works for closed surfaces*
2. Solves the word problem
3. Approximates stretch factors
4. Computes the order
Macaw (implementation)

1. *Works for closed surfaces*
2. Solves the word problem
3. Approximates stretch factors
4. Computes the order

Contributors are welcome!
Macaw (implementation)

1. *Works for closed surfaces*
2. Solves the word problem
3. Approximates stretch factors
4. Computes the order

Contributors are welcome!

Thank you!
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Adding points to configurations

Lei Chen
University of Chicago
The general problem
The general problem

\[ \text{Forget} \]

\[ \text{Conf}_{n+1}(M) \quad \text{Forget} \quad \text{Conf}_n(M) \]
The general problem

\[ \text{Conf}_{n+1}(M) \]

Forget

? 

\[ \text{Conf}_n(M) \]
The general problem

\[ \text{Conf}_{n+1}(M) \]

\[ \text{Forget} \]

\[ \text{Conf}_n(M) \]
The general problem

\[ \text{Conf}_{n+1}(M) \]

Forget

\[ \text{Conf}_n(M) \]

new point
Example 1

$$\text{Conf}_4(S^2) \xrightarrow{\text{Forget}} \text{Conf}_3(S^2)$$
Example 1

$$\text{Conf}_4(S^2) \xrightarrow{\text{Forget}} \text{Conf}_3(S^2)$$
Example 1

\[ \text{Conf}_4(S^2) \xrightarrow{\text{Forget}} \text{Conf}_3(S^2) \]
Example 1

\[ \text{Conf}_4(S^2) \xrightarrow{\text{Forget}} \text{Conf}_3(S^2) \]

\[ A \in PSL_2(\mathbb{C}) \]
Example 1

\[ \text{Conf}_4(S^2) \xrightarrow{\text{Forget}} \text{Conf}_3(S^2) \]

Diagram:
- \( A^{-1}(i) \)
- \( A \in PSL_2(\mathbb{C}) \)
Example 2: $\mathbb{R}^2$
Example 2: $\mathbb{R}^2$

Add far away
Example 2: $\mathbb{R}^2$

Add far away

New point
Example 2: $\mathbb{R}^2$

Add far away

Add close by

New point

vector field
Example 2: $\mathbb{R}^2$

Add far away

Add close by vector field

New point
Theorem (C-)
Theorem (C-)
When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \exists \text{Conf}_n(M)$?
When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M)$?

1) $n > 3, M = \mathbb{R}^2$
Theorem (C-) When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M)$?

1) $n > 3, M = \mathbb{R}^2$ Only “Add far away”

“Add close by”
Theorem (C-)

When does \( \text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M) \) forget \( \exists \)?

1) \( n > 3, M = \mathbb{R}^2 \)

2) \( M = S^2 \)

Only “Add far away”
“Add close by”
Theorem (C-)

When does \( \text{Conf}_{n+1}(M) \xrightarrow{\text{for} \exists} \text{Conf}_n(M) \)?

1) \( n > 3, M = \mathbb{R}^2 \)

2) \( M = S^2 \)

Only “Add far away”

“No for \( n=2 \)”

“Add close by”
Theorem (C-)

When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M)$?

1) $n > 3, \ M = \mathbb{R}^2$

2) $M = S^2$

Only “Add far away”

“Add close by”

No for $n=2$

Yes for $n>2$
Theorem (C-)

When does $\text{Conf}_{n+1}(M) \xrightarrow{\exists} \text{Conf}_n(M)$?

1) $n > 3, M = \mathbb{R}^2$

2) $M = S^2$

Only "Add far away"
"Add close by"

Yes for $n > 2$

No for $n = 2$

For $n > 4$, only "Add close by"
Theorem (C-)

When does \( \text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M) \)?

1) \( n > 3, M = \mathbb{R}^2 \)

2) \( M = S^2 \)
   - No for \( n=2 \)
   - Yes for \( n>2 \)

3) \( g > 1, M = S_g \)

Only "Add far away"
"Add close by"

For \( n>4 \),
only "Add close by"
Theorem (C-)

When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M)$?

1) $n > 3$, $M = \mathbb{R}^2$

- Only "Add far away"
- "Add close by" for $n > 4$

2) $M = S^2$

- No for $n=2$
- Yes for $n > 2$

3) $g > 1$, $M = S_g$

- No for $n > 1$
- For $n > 4$, only "Add close by"
When does $\text{Conf}_{n+1}(M) \xrightarrow{\text{forget}} \text{Conf}_n(M)$? 

1) $n > 3, M = \mathbb{R}^2$ 
   Only "Add far away"
   "Add close by"

2) $M = S^2$ 
   Only "Add far away"
   "Add close by"
   
   Yes for $n > 2$
   For $n > 4$, 
   only "Add close by"

3) $g > 1, M = S_g$ 
   No for $n = 2$

   No for $n > 1$

   Yes for $n = 1$
Methods used
Methods used

Thurston’s normal form

Thurston, Birman-Lubotzky-McCarthy, Ivanov and others
Methods used

Thurston’s normal form

Thurston, Birman-Lubotzky-McCarthy, Ivanov and others

Obstructions in $H^*(\text{Conf}_n(M); \mathbb{Q})$
Problem: Other M?
Problem: Other $M$?

\[ \text{Conf}_3(\mathbb{C}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{C}P^2) \]

\[ \text{Conf}_3(\mathbb{R}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{R}P^2) \]
Problem: Other M?

Conf_3(\mathbb{C}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{C}P^2)

Conf_3(\mathbb{R}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{R}P^2)
Problem: Other M?

Cross product

\[ \text{Conf}_3(\mathbb{C}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{C}P^2) \]

Cross product

\[ \text{Conf}_3(\mathbb{R}P^2) \xrightarrow{\text{forget}} \text{Conf}_2(\mathbb{R}P^2) \]

Do we have other examples like these? Exotic sections?
Reference:

Pre-print: Section problems for configuration spaces of surfaces
https://arxiv.org/abs/1708.07921

Thanks!!
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION
Mapping class groups: Bigger. Better? Commensurably rigid!

Spencer Dowdall
(with Juliette Bavard & Kasra Rafi)

Vanderbilt University
math.vanderbilt.edu/dowdalsd/

No boundaries conference
October 28, 2017
$S$ – oriented surface without boundary

$$\text{Mod}(S) = \text{Homeo}(S)/\text{isotopy}$$
Mapping class groups

$S$ – oriented surface without boundary

$$\text{Mod}(S) = \text{Homeo}(S)/\text{isotopy}$$
$S$ – oriented surface without boundary

$$\text{Mod}(S) = \text{Homeo}(S)/\text{isotopy}$$
Go Bigger!

**Big mapping class groups:** \( \text{Mod(} \text{infinite-type surface} \text{)} \)
Big mapping class groups: $\text{Mod}(\text{infinite-type surface})$

Sphere minus Cantor set
Go Bigger!

**Big mapping class groups:** $\text{Mod}(\text{infinite-type surface})$

Sphere minus Cantor set

Loch Ness Monster surface

Uncountable $\text{Mod}(S)$ inherits nondiscrete topology from $\text{Homeo}(S)$

Big mapping class groups: $\text{Mod}(\text{infinite-type surface})$

Sphere minus Cantor set

Loch Ness Monster surface

Here be dragons!
- uncountable
- $\text{Mod}(S)$ inherits nondiscrete topology from $\text{Homeo}(S)$
Recognition and Rigidity

**Question:** Do big mapping class groups distinguish surfaces?
Question: Do big mapping class groups distinguish surfaces? **YES!**

**Theorem (Bavard–D.–Rafi)**

$S_1$ and $S_2$ infinite-type surfaces. Any isomorphism $G_1 \to G_2$ between finite index subgroups $G_i$ of $\text{Mod}(S_i)$ is induced by a homeomorphism $S_1 \to S_2$. 

**Corollary**

For $S$ an infinite-type surface:

$$\text{Aut}(\text{Mod}(S)) \hookrightarrow \text{Mod}(S) \hookrightarrow \text{Comm}(\text{Mod}(S)).$$
Recognition and Rigidity

**Question:** Do big mapping class groups distinguish surfaces? **YES!**

**Theorem (Bavard–D.–Rafi)**

$S_1$ and $S_2$ infinite-type surfaces. Any isomorphism $G_1 \to G_2$ between finite index subgroups $G_i$ of $\text{Mod}(S_i)$ is induced by a homeomorphism $S_1 \to S_2$.

**Corollary**

*For $S$ an infinite-type surface:*

$$\text{Aut}(\text{Mod}(S)) \cong \text{Mod}(S)$$
**Recognition and Rigidity**

**Question:** Do big mapping class groups distinguish surfaces? **YES!**

**Theorem (Bavard–D. –Rafi)**

$S_1$ and $S_2$ infinite-type surfaces. Any isomorphism $G_1 \to G_2$ between finite index subgroups $G_i$ of $\text{Mod}(S_i)$ is induced by a homeomorphism $S_1 \to S_2$.

**Corollary**

For $S$ an infinite-type surface:

$$\text{Aut}(\text{Mod}(S)) \cong \text{Mod}(S) \cong \text{Comm}(\text{Mod}(S))$$
Question: Do big mapping class groups distinguish surfaces? **YES!**

**Theorem (Bavard–D.–Rafi)**

$S_1$ and $S_2$ infinite-type surfaces. Any isomorphism $G_1 \to G_2$ between finite index subgroups $G_i$ of $\text{Mod}(S_i)$ is induced by a homeomorphism $S_1 \to S_2$.

**Corollary**

For $S$ an infinite-type surface:

$$\text{Aut}(\text{Mod}(S)) \cong \text{Mod}(S) \cong \text{Comm}(\text{Mod}(S))$$

**Abstract Commensurator**

$\text{Comm}(G) =$ group of isomorphisms between finite-index subgroups

- E.g: $\text{Aut}(\mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z}$, but $\text{Comm}(\mathbb{Z}) = \mathbb{Q}^*$
Ingredients

1) Algebraic identification of Dehn twists:

Key Lemma (Bavard–D.–Rafi)

\[ f \in \text{Mod}(S) \text{ has finite-support } \iff f \text{'s conjugacy class is countable.} \]
Ingredients

1) Algebraic identification of Dehn twists:

Key Lemma (Bavard–D.–Rafi)

\[ f \in \text{Mod}(S) \text{ has finite-support } \iff f \text{'s conjugacy class is countable.} \]

2) Rigidity of curve complexes \( C(S) \):

Theorem (Hernandez–Morales–Valdez; Bavard–D.–Rafi)

\[ S_1 \text{ and } S_2 \text{ infinite-type surfaces. Any automorphism } C(S_1) \to C(S_2) \text{ of their curve complexes is induced by a homeomorphism } S_1 \to S_2. \]
NO BOUNDARIES
LIGHTNING TALKS
SATURDAY SESSION