1. Suppose that $A$ is a $2 \times 2$ matrix with non-negative real entries. Show that $A$ commutes with its transpose (i.e. $A^t$) under multiplication if and only if $A = A^t$.

2. Compute the determinant of the following matrix

$$
\begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 6 & 9 \\
3 & 6 & 10 & 12 \\
4 & 9 & 12 & 16
\end{bmatrix}
$$

3. Suppose that $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a linear transformation and $v \in \mathbb{R}^n$ is some vector.
   a. Fix $v \in \mathbb{R}^n$. Show that $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ given by

$$
f(x) = T(x) + v
$$

is a linear transformation if and only if $v = 0$.

   b. Mappings of the type $f$ just considered are called “affine maps”. Let $A(\mathbb{R}^n, \mathbb{R}^n)$ denote the space of all affine maps, where if $f, g$ are two such maps, with associated transformations $T_1$ and $T_2$ and vectors $v_1$ and $v_2$, then the mapping $f + g$ is defined to be

$$
(f + g)(x) = (T_1 + T_2)(x) + (v_1 + v_2) = T_1(x) + T_2(x) + v_1 + v_2.
$$

And scalar multiplication is defined similarly. Determine

$$
\dim(A(\mathbb{R}^n, \mathbb{R}^n)).
$$
4. Use Gaussian elimination to solve the system

\[
\begin{align*}
x + 2y + 3z + 4w &= 1 \\
2x + 5y + 6z + 10w &= 3 \\
3x + 7y + 9z + 15w &= 5.
\end{align*}
\]

5. Define the following terms.
   a. linear transformation \( f : \mathbb{R}^n \to \mathbb{R}^m \).
   b. nullity.
   c. cofactor of an \( n \times n \) matrix \( A \).
   d. surjective map.
   e. nullspace.