1. Suppose that $f : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation that maps each subspace of dimension 2 one-to-one to some other subspace of dimension 2. Show that the only vector $v$ for which $f(v) = 0$ is $v = 0$.

2. Suppose that $X$ is a subspace of a vector space $V$. Prove that the orthogonal complement of the orthogonal complement of $X$ is $X$.

3. Prove that the cross product is non-associative, as well as non-commutative.

4. Fix a vector $x \in \mathbb{R}^3$. Consider the set of all pairs $P$ of vectors $(v, w) \in \mathbb{R}^3 \times \mathbb{R}^3$ satisfying $v \times (x \times w) = 0$. Is $P$ a subspace of $\mathbb{R}^3 \times \mathbb{R}^3$?

5. Suppose that $P_1$ and $P_2$ are two planes in $\mathbb{R}^3$, and let $L$ be a line in $\mathbb{R}^3$ that hits both $P_1$ and $P_2$. Suppose that $|L \cap P_1| + |L \cap P_2| \geq 3$. Prove that $P_1 \cap P_2$ contains a line.

6. For which triples of vectors $u, v, w$ is the scalar triple product $[u, v, w] > 0$? (Describe the triples.)
7. Suppose that \( A, B, C \) are elements of some vector space \( V \). Further, suppose that
\[
\]
Is it the case that \( A, B, C \) are all mutually orthogonal? What about if you just use two vectors \( A, B \) (i.e. does \( ||A + B||^2 = ||A||^2 + ||B||^2 \) imply \( A \) and \( B \) orthogonal?)?

8. What is the difference between normal and orthogonal?

9. Suppose that \( X \) is a subspace of a finite dimensional vector space \( V \).
   a. Prove that \( X \) is also finite dimensional.
   b. Prove that \( X^\perp \) is finite dimensional.

10. Fix a vector space \( V \). Do the set of subspaces of \( V \) form a vector space? Here, addition of subspaces is defined as follows: Given subspaces \( A, B \), we have
\[
A + B = \{ a + b : a \in A, b \in B \}.
\]
And, scalar multiplication is
\[
\lambda A := \{ \lambda a : a \in A \}.
\]