Through a Glass Darkly

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1 Prolegomena

Education is a repetition of civilization in little.

— Herbert Spencer

Being a mathematician is like being a manic depressive. One experiences occasional moments of giddy elation, interwoven with protracted periods of black despair. Yet this is the life path that we choose for ourselves. And we wonder why nobody understands us.

The budding mathematician spends an extraordinarily long period of study and backbreaking hard work in order to attain the Ph.D. And that is only an entry card into the profession. It hardly makes one a mathematician.

To be able to call oneself a mathematician, one must have proved some good theorems and written some good papers thereon. One must have given a number of talks on his work, and (ideally) one should have either an academic job or a job in the research infrastructure. Then, and only then, can one hold one's head up in the community and call oneself a peer of the realm. Often

one is thirty years old before this comes about. It is a protracted period of apprenticeship, and there are many fallen and discouraged and indeed lost along the way.

The professional mathematician spends his life thinking about problems that he cannot solve, and learning from his (repeated and often maddening) mistakes. That he can very occasionally pull the fat out of the fire and make something worthwhile of it is in fact a small miracle. And even when he can pull off such a feat, what are the chances that his peers in the community will toss their hats in the air and proclaim him a hail fellow well met? Slim to none at best.

In the end we learn to do mathematics because of its intrinsic beauty, and its enduring value, and for the personal satisfaction it gives us. It is an important, worthwhile, dignified way to spend one's time, and it beats almost any other avocation that I can think of. But it has its frustrations.

There are few outside of the mathematical community who have even the vaguest notion of what we do, or how we spend our time. Surely they have no sense of what a theorem is, or how one proves a theorem, or why one would want to.[2] How could one spend a year or two studying other people's work, only so that one can spend yet several more years to develop one's own work? Were it not for tenure, how could any mathematics ever get done?

We in the mathematics community expect (as we should) the state legislature to provide funds for the universities (to pay our salaries, for instance). We expect the members of Congress to allocate funds for the National Science Foundation and other agencies to subvent our research. We expect the White House Science Advisor to speak well of academics, and of mathematicians in particular, so that we can live our lives and enjoy the fruits of our labors. But what do these people know of our values and our goals? How can we hope that, when they do the obvious and necessary ranking of priorities that must be a part of their jobs, we will somehow get sorted near the top

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[2]From my solipsistic perspective as a mathematician, this is truly tragic. For mathematical thinking is at the very basis of human thought. It is the key to an examined life.

of the list?

This last paragraph explains in part why we as a profession can be aggravated and demoralized, and why we endure periods of frustration and hopelessness. We are not by nature articulate—especially at presenting our case to those who do not speak our language—and we pay a price for that incoherence. We tend to be solipsistic and focused on our scientific activities, and trust that the value of our results will speak for themselves. When competing with the `Wii` and the `iPod`, we are bound therefore to be daunted.

2 Life in the Big City

> The most savage controversies are about those matters
> as to which there is no good evidence either way.
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> — Bertrand Russell

If you have ever been Chair of your department, put in the position of explaining to the Dean what the department's needs are, you know how hard it is to explain our mission to the great unwashed. You waltz into the Dean's office and start telling him how we must have someone in Ricci flows, we certainly need a worker in mirror symmetry, and what about that hot new stuff about the distribution of primes using additive combinatorics? The Dean, probably a chemist, has no idea what you are talking about.

Of course the person who had the previous appointment with the Dean was the Chair of Chemistry, and he glibly told the Dean how they are woefully shy of people in radiochemistry and organic chemistry. And an extra physical chemist or two would be nice as well. The Dean said "sure", he understood immediately. It was a real shift of gears then for the Dean to have to figure out what in the world you (from the Mathematics Department) are talking about. How do you put your case in words that the Dean will understand?

How do you sell yourself (and your department) to him?[3]

Certainly we have the same problem with society at large. People understand, just because of their social milieu, why medicine is important and useful. Computers and their offspring make good sense; we all encounter computers every day and have at least a heuristic sense of what they are good for. Even certain parts of engineering resonate with the average citizen (aeronautics, biomedical engineering, civil engineering). But, after getting out of school, most people have little or no use for mathematics. Most financial transactions are handled by machines. Most of us bring our taxes to professionals for preparation. Most of us farm out construction projects around the house to contractors. If any mathematics, or even arithmetic, is required in the workplace it is probably handled by software.

One of my wife's uncles, a farmer, once said to me—thinking himself to be in a puckish mood—that we obviously no longer need mathematicians because we have computers. I gave him a patient look and said yes, and we obviously no longer need farmers because we have vending machines. He was not amused. But the analogy is a good one. Computers are great for manipulating data, but not for thinking. Vending machines are great for handing you a morsel of food *that someone else has produced in the traditional fashion.*

People had a hard time understanding what Picasso's art was about—or even Andy Warhol's art—but they had a visceral sense that it was interesting and important. The fact that people would spend millions of dollars for the paintings gave the activity a certain *gravitas*, but there is something in the nature of art that makes it resonate with our collective unconscious. With mathematics, people spend their lives coming to grips with what was likely a negative experience in school, reinforced by uninspiring teachers and dreadful textbooks. If you are at a cocktail party and announce that you don't like art, or don't like music, people are liable to conclude that you are some

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[3]It is arguable that a mathematics department is better off with a Dean who is a musicologist or perhaps a philologist. Such a scholar is not hampered by the *Realpolitik* of lab science dynamics, and can perhaps think imaginatively about what our goals are.

kind of philistine. If instead you announce that you don't like mathematics, people conclude that you are a regular guy. [If you choose to announce that you *do* like mathematics, people are liable to get up and walk away.] To the uninitiated, mathematics is cold and austere and unforgiving. It is difficult to get even an intuitive sense of what the typical mathematician is up to. Unlike physicists and biologists (who have been successfully communicating with the press and the public for more than fifty years), we are not good at telling half-truths so that we can paint a picture of our meaning and get our point across. We are too wedded to the mathematical method. We think in terms of definitions and axioms and theorems.

3   Living the Good Life

> One normally thinks that everything that is true is true for a reason. I've found mathematical truths that are true for no reason at all. These mathematical truths are beyond the power of mathematical reasoning because they are accidental and random.
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> — G. J. Chaitin

The life of a mathematician is a wonderful experience. It is an exhilarating, blissful existence for those who are prone to enjoy it. One gets to spend one's time with like-minded people who are in pursuit of a holy grail that is part of an important and valuable larger picture that we are all bound to. One gets to travel, and spend time with friends all over the world, and hang out in hotels, and eat exotic foods, and drink lovely drinks. One gets to teach bright students and engage in the marketplace of ideas, and actually to develop new ones. What could be better? There is hardly a more rewarding way to be professionally engaged.

It is a special privilege to be able to spend one's time—and be paid for it—thinking original (and occasionally profound) thoughts and developing new programs and ideas. One actually feels that he is changing the fabric of the cosmos, helping people to see things that they have not seen before, affecting people's lives.[4] Teaching can and probably should be a part of this process. For surely bringing along the next generation, training a new flank of scholars, is one of the more enlightened and certainly important pursuits. Also interacting with young minds is a beautiful way to stay vibrant and plugged in, and to keep in touch with the development of new ideas.

Of course there are different types of teaching. The teaching of rudimentary calculus to freshmen has different rewards from teaching your latest research ideas to graduate students. But both are important, and both yield palpable results. What is more, *this is an activity that others understand and appreciate.* If the public does not think of us in any other way, surely they think of us as teachers. And better that *we* should have to do it. After all, it is our bailiwick.

The hard fact of the matter is that the powers that be in the university also appreciate our teaching rather more than they do our many other activities. After all, mathematics is a key part of the core curriculum. A university could hardly survive without mathematics. Other majors could not function, could not advance their students, could not build their curricula, without a basis in mathematics. So our teaching role at the institution is both fundamental and essential. Our research role is less well understood, especially because *we do not by instinct communicate naturally with scholars in other departments.*

This is actually a key point. We all recall the crisis at the University of Rochester thirteen years ago, when the Dean shut down the graduate program in mathematics. His reasoning, quite simply, was that he felt that the mathematics department was isolated, did not interact productively with

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[4]I have long been inspired by Freeman Dyson's book [DYS]. It describes both poignantly and passionately the life of the scientist, and how he can feel that he is altering and influencing the world around him.

other units on campus, did not carry its own weight. The event at Rochester rang a knell throughout the profession, for we all knew that similar allegations could be leveled at any of us. Institutions like Princeton or Harvard are truly ivory towers, and unlikely to suffer the sort of indignity being described here. But if you work at a public institution then look out. I work at a *very* private university, and I can tell you that, in my negotiations as Chair with our Dean, he sometimes brought up Rochester. And he did *not* do so in an effort to be friendly. He was in fact threatening me.

Some departments, like Earth & Planetary Science or Biomedical Engineering, interact very naturally with other subjects. Their material is intrinsically interdisciplinary. It makes perfect sense for these people to develop cross-disciplinary curricula and joint majors with other departments. It is very obvious and sensible for them to apply for grants with people from departments even outside of their School. A faculty member of such a department will speak several languages fluently.

It is different for mathematics. It is a challenge just to speak the one language of mathematics, and to speak it well. Most of us do a pretty good job at it, and those outside of mathematics cannot do it at all. So there is a natural barrier to communication and collaboration. In meetings with other faculty—even from physics and engineering—we find difficulty identifying a common vocabulary. We find that we have widely disparate goals, and very different means of achieving them.

Also our value systems are different. Our methods for gauging success vary dramatically. Our reward systems deviate markedly. Once you become a full Professor you will serve on tenure and promotion committees for other departments. This experience is a real eye-opener, for you will find that the

criteria used in English and History and Geography are quite different from what we are accustomed to.[5] Even our views of truth can be markedly different.

4   The Why and the Wherefore

> The lofty light of the a priori outshines the dim light of the world and makes for us incontrovertible truths because of their "clearness and distinctness."
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> — René Descartes

A mathematician typically goes through most of his early life as a flaming success at everything he does. One excels in grade school, one excels in high school, one excels in college. Even in graduate school one can do quite well if one is willing to put forth the effort.

Put in slightly different terms: One can get a long way in the basic material just by being smart. Not so much effort or discipline is required. And this may explain why so many truly brilliant people get left in the dust. They reach a point where some real *Sitzfleisch* and true effort are required, and they are simply not up to it. They have never had to expend such disciplined study before, so why start now?

While there is no question that being smart can take one a long way, there comes a point—for *all of us*—where it becomes clear that a capacity for hard work can really make a difference. Most professional mathematicians put in *at least* ten hours per day, *at least* six days per week. There are many who do much more. And we tend to enjoy it. The great thing about mathematics is

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[5]I still recall serving on the committee for promotion to Professor of a candidate in Geography. One of his published writings was called *A Walk Through China Town*. It described the experience of walking down Grant Avenue in San Francisco and smelling the wonton soup. What would be the analogue of this in a case for promotion in Mathematics?

that it does not fight you. It will not sneak behind your back and bite you. It is always satisfying and always rewarding.

Doing mathematics is *not* like laying bricks or mowing the grass. The quantity of end product is not a linear function of the time expended. Far from it. As Charles Fefferman, Fields Medalist, once said, a good mathematician throws 90% of his work in the trash. Of course one learns from all that work, and it makes one stronger for the next sortie. But one often, at the end of six months or a year, does not have much to show.

On the other hand, one can be blessed with extraordinary periods of productivity. The accumulated skills and insights of many years of study suddenly begin to pay off, and one finds that he has plenty to say. And it is *quite* worthwhile. Certainly worth writing up and sharing with others and publishing. This is what makes life rewarding, and this is what we live for.

Economists like to use professors as a model, because they run contrary to many of the truisms of elementary economic theory. For example, if you pay a Professor of Mathematics twice as much, that does not mean that he will be able to prove twice as many theorems, or produce twice as many graduate students. The truth is that he is probably already working to his capacity. There are only so many hours in the day. What more could he do? It is difficult to say what a Professor of Mathematics should be compensated, because we do not fit the classical economic model.

Flipped on its head, we could also note that if you give a Professor of Mathematics twice as much to do, it does not follow that he will have a nervous breakdown, or quit, or go into open rebellion. Many of us now have a teaching load of two courses per semester. But sixty years ago the norm—even at the very best universities in the United States—was three courses (or more!) per semester. Also, in those days, there was very little secretarial help. Professors did a lot of the drudgery themselves. There were also no NSF grants, and very little discretionary departmental money, so travel was often subvented from one's own pocket. Today life is much better for everyone.

The fact is that a Professor of Mathematics has a good deal of slack built into his schedule. If you double his teaching load, it means that he has less time to go to seminars, or to talk to his colleagues, or just to sit and think. But he will still get through the day. Just with considerably less enthusiasm. And notably less creativity. Universities are holding faculty much more accountable for their time these days. Total Quality Management is one of many insidious ideas from the business world that is starting to get a grip at our institutions of higher learning. In twenty years we may find that we are much more like teachers (in the way that we spend our time) and much less like scholars.

Sad to say, the Dean or the Provost has only the vaguest sense of what our scholarly activities are. When they think of the math department at all, they think of us as "those guys who teach calculus." They certainly *do not* think of us as "those guys who proved the Bieberbach conjecture." Such a statement would have little meaning for the typical university administrator. Of course they are pleased when the faculty garners kudos and awards, but the awards that Louis de Branges received for his achievement were fairly low key.[6] They probably would not even raise an eyebrow among the Board of Trustees.

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[6] When I was Chair of the Mathematics Department, the Dean was constantly reminding me that he thought of us as a gang of incompetent, fairly uncooperative boobs. One of his very favorite Chairs at that time was the Head of Earth & Planetary Sciences. This man was in fact the leader of the Mars space probe team, and he actually designed the vehicle that was being used to explore Mars. Well, you can imagine the kind of presentations that this guy could give—lots of animated graphics, lots of panoramic vistas, lots of dreamy speculation, lots of stories about other-worldly adventures. His talks were given in the biggest auditoriums on campus, and they were always packed. The Dean was front and center, with his tongue hanging out, every time; he fairly glowed in the dark because he was so pleased and excited. How can a mathematician compete with that sort of showmanship? Even if I were to prove the Riemann Hypothesis, it would pale by comparison.

5 Such is Life

There is no religious denomination in which the misuse of metaphysical expressions has been responsible for so much sin as it has in mathematics.

— Ludwig Wittgenstein

Mathematicians are very much like oboe players. They do something quite difficult that nobody else understands. That is fine, but it comes with a price.

We take it for granted that we work in a rarified stratum of the universe that nobody else will understand. We do not expect to be able to communicate with others. When we meet someone at a cocktail party and say, "I am a mathematician," we expect to be snubbed, or perhaps greeted with a witty rejoinder like, "I was never any good in math." Or, "I was good at math until we got to that stuff with the letters—like algebra."

When I meet a brain surgeon I never say, "I was never any good at brain surgery. Those lobotomies always got me down." When I meet a proctologist, I am never tempted to say, "I was never any good at ...." Why do we mathematicians elicit such foolish behavior from people?

One friend of mine suggested that what people are really saying to us, when they make a statement of the sort just indicated, is that they spent their college years screwing around. They never buckled down and studied anything serious. So now they are apologizing for it. This is perhaps too simplistic. For taxi drivers say these foolish things too. And so do mailmen and butchers. Perhaps what people are telling us is that they *know* that they should understand and appreciate mathematics, but they do not. So instead they are resentful.

There is a real disconnect when it comes to mathematics. Most people, by the time that they get to college, have had enough mathematics so that

they can be pretty sure they do not like it. They certainly do not want to major in the subject, and their preference is to avoid it as much as possible. Unfortunately, for many of these folks, their major may require a nontrivial amount of math (not so much because the subject area actually *uses* mathematics, but rather because the people who run the department seem to want to use mathematics as a *filter*). And also unfortunately it happens, much more often than it should, that people end up changing their majors (from engineering to psychology or physics to media studies) simply because they cannot hack the math.

In recent years I have been collaborating with plastic surgeons, and I find that this is a wonderful device for cutting through the sort of conversational impasse that we have been describing. *Everyone*, at least everyone past a certain age, is quite interested in plastic surgery. People want to understand it, they want to know what it entails, they want to know what are the guarantees of success. When they learn that there are connections between plastic surgery and mathematics then that is a hint of a human side of math. It gives me an entree that I never enjoyed in the past.

I also once wrote a paper with a picture of the space shuttle in it. That did not prove to be quite so salubrious for casual conversation; after all, engineering piled on top of mathematics does not make the mathematics any more palatable. But at least it was an indication that I could speak several tongues.

And that is certainly a point worth pondering if we want to fit into a social milieu. Speaking many tongues is a distinct advantage, and gives one a wedge for making real contact with people. It provides another way of looking at things, a new point of contact. Trying to talk to people *about mathematics*, *in the language of mathematics*, *using the logic of mathematics* is not going to get you very far. It will not work with newspaper reporters and it also will not work with ordinary folks that you are going to meet in the course of your life.

6   Mathematics and Art

It takes a long time to understand nothing.

— Edward Dahlberg

Even in the times of ancient Greece there was an understanding that mathematics and art were related. Both disciplines entail symmetry, order, perspective, and intricate relationships among the components. The golden mean is but one of many artifacts of this putative symbiosis.

M. C. Escher spent a good deal of time at the Moorish castle the Alhambra, studying the very mathematical artwork displayed there. This served to inspire his later studies (which are considered to be a very remarkable synthesis of mathematics and art).

Today there is more formal recognition of the interrelationship of mathematics and art. No less an eminence than Louis Vuitton offers a substantial prize each year for innovative work on the interface of mathematics and art. Benoit Mandelbrot has received this prize (for his work on fractals—see [MAN]), and so has David Hoffman for his work with Jim Hoffman and Bill Meeks on embedded minimal surfaces (see [HOF]).

Mathematics and art make a wonderful and fecund pairing for, as we have discussed here, mathematics is perceived in general to be austere, unforgiving, cold, and perhaps even lifeless. By contrast, art is warm, human, inspiring, even divine. If I had to give an after-dinner talk about what I do, I would not get very far trying to discuss the automorphism groups of pseudoconvex domains. I would probably have much better luck discussing the mathematics in the art of M. C. Escher, or the art that led to the mathematical work of Celso Costa on minimal surfaces.

Of course we as mathematicians perceive our craft to be an art form. Those among us who can see—and actually prove!—profound new theorems are held in the greatest reverence, much as artists. We see the process of

divining a new result and then determining how to verify it much like the process of eking out a new artwork. It would be in our best interest to convey this view of what we do to the world at large. Whatever the merits of fractal geometry may be, Benoit Mandelbrot has done a wonderful job of conveying both the art and the excitement of mathematics to the public.

Those who wish to do so may seek mathematics exhibited in art throughout the ages. Examples are

- A marble mosaic featuring the small stellated dodecahedron, attributed to Paolo Uccello, in the floor of the San Marco Basilica in Venice.

- Leonardo da Vinci's outstanding diagrams of regular polyhedra drawn as illustrations for Luca Pacioli's book *The Divine Proportion*.

- A glass rhombicuboctahedron in Jacopo de' Barbari's portrait of Pacioli, painted in 1495.

- A truncated polyhedron (and various other mathematical objects) which feature in Albrecht Dürer's engraving Melancholia I.

- Salvador Dalí's painting *The Last Supper* in which Christ and his disciples are pictured inside a giant dodecahedron.

Sculptor Helaman Ferguson [FER] has made sculptures in various materials of a wide range of complex surfaces and other topological objects. His work is motivated specifically by the desire to create visual representations of mathematical objects. There are many artists today who conceive of themselves, and indeed advertise themselves, as mathematical artists. There are probably rather fewer mathematicians who conceive of themselves as artistic mathematicians.

Mathematics and music have a longstanding and deeply developed relationship. Abstract algebra and number theory can be used to understand musical structure. There is even a well-defined subject of musical set theory (although it is used primarily to describe atonal pieces). Pythagorean tuning

is based on the perfect consonances. Many mathematicians are musicians, and take great comfort and joy from musical pastimes. Music can be an opportunity for mathematicians to interact meaningfully with a broad cross section of our world. Mathematicians Noam Elkies and David Wright have developed wonderful presentations—even full courses—about the symbiosis between mathematics and music.

Mathematics can learn a lot from art, especially from the way that art reaches out to humanity. Part of art is the interface between the artist and the observer. Mathematics is like that too, but typically the observer is another mathematician. We would do well, as a profession, to think about how to expand our pool of observers.

7   Mathematics vs. Physics

I do still believe that rigor is a relative notion, not an absolute one. It depends on the background readers have and are expected to use in their judgment.

— René Thom

Certainly "versus" is the wrong word here. Ever since the time of Isaac Newton, mathematics and physics have been closely allied. After all, Isaac Newton virtually invented physics as we know it today. And mathematics in his day was a free-for-all. So the field was open for Newton to create any synthesis that he chose.

But mathematics and physics are divided by a common goal, which is to understand the world around us. Physicists perceive that "world" by observing and recording and thinking. Mathematicians perceive that "world" by looking within themselves (but see the next section on Platonism vs. Kantianism).

And thus arises a difference in styles. The physicist thinks of himself as an observer, and is often content to describe what he sees. The mathematician is *never* so content. Even when he "sees" with utmost clarity, the mathematician wants to confirm that vision with a proof. This fact makes us precise and austere and exacting, but it also sets us apart and makes us mysterious and difficult to deal with.

I once heard Fields Medalist Charles Fefferman give a lecture (to a mixed audience of mathematicians and physicists) about the existence of matter. In those days Fefferman's goal was to prove the existence of matter from first principles—in an axiomatic fashion. I thought that this was a fascinating quest, and I think that some of the other mathematicians in the audience agreed with me. But at some point during the talk a frustrated physicist raised his hand and shouted, "Why do you need to do this? All you have to do is look out the window to see that matter exists!"

Isn't it wonderful? Different people have different value systems and different ways to view the very same scientific facts. If there is a schism between the way that mathematicians view themselves and the way that *physicists* see us, then there is little surprise that there is such a schism between our view of ourselves and the way that non-scientists see us. Most laymen are content to accept the world phenomenologically—it is what it is. Certainly it is not the average person's job to try to dope out why things are the way they are, or who made them that way. This all borders on theology, and that is a distinctly painful topic. Better to go have a beer and watch a sporting event on the large-screen TV. This is *not* the view that a mathematician takes.

The world of the mathematician is a world that we have built for ourselves. And it makes good sense that we have done so, for we need this infrastructure in order to pursue the truths that we care about. But the nature of our subject also sets us apart from others—even from close allies like the physicists. We not only have a divergence of points of view, but also an impasse in communication. We often cannot find the words to enunciate

what we are seeing, or what we are thinking.

In fact it has taken more than 2500 years for the modern mathematical mode of discourse to evolve. Although the history of proof is rather obscure, we know that the efforts of Thales and Protagoras and Hippocrates and Theaetetus and Plato and Pythagoras and Aristotle, culminating in Euclid's magnificent *Elements*, have given us the axiomatic method and the language of proof. In modern times, the work of David Hilbert and Nicolas Bourbaki have helped us to sharpen our focus and nail down a universal language and methodology for mathematics (see [KRA] for a detailed history of these matters and for many relevant references). The idea of mathematical proof is still changing and evolving, but it is definitely part of who we are and what we believe.

The discussion of Platonism and Kantianism in the next section sheds further light on these issues.

8   Plato vs. Kant

It is by logic we prove, it is by intuition that we invent.

— Henri Poincaré

A debate has been festering in the mathematics profession for a good time now, and it seems to have heated up in the past few years (see, for instance [DAV]). And the debate says quite a lot about who we are and how we endeavor to think of ourselves. It is the question of whether our subject is Platonic or Kantian.

The Platonic view of the world is that mathematical facts have an independent existence—very much like classical Platonic ideals—and the research mathematician *discovers* those facts—very much like Amerigo Vespucci discovered America, or Jonas Salk discovered his polio vaccine. But it should

be clearly understood that, in the Platonic view, mathematical ideas exist in some higher realm that is independent of the physical world, and certainly independent of any particular person. Also independent of time. The Platonic view poses the notion that a theorem can be "true" before it is proved.

The Kantian view of the world is that the mathematician creates the subject from within himself. The idea of set, the idea of group, the idea of pseudoconvexity, are all products of the human mind. They do not exist out there in nature. We (the mathematical community) have *created* them.

My own view is that both these paradigms are valid, and both play a role in the life of any mathematician. On a typical day, the mathematician goes to his office and sits down and thinks. He will certainly examine mathematical ideas that already exist, and can be found in some paper penned by some other mathematician. But he will also cook things up from whole cloth. Maybe create a new axiom system, or define a new concept, or formulate a new hypothesis. These two activities are by no means mutually exclusive, and they both contribute to the rich broth that is mathematics.

Of course the Kantian position raises interesting epistemological questions. Do we think of mathematics as being created by each individual? If that is so, then there are hundreds if not thousands of distinct individuals creating mathematics from within. How can they communicate and share their ideas? Or perhaps the Kantian position is that mathematics is created by some shared consciousness of the aggregate humanity of mathematicians. And then is it up to each individual to "discover" what the aggregate consciousness has been creating? Which is starting to sound awfully Platonic. Saunders Mac Lane [MAC] argues cogently that mathematical ideas are elicited or abstracted from the world around is. This is perhaps a middle path between the two points of view.

The Platonic view of reality seems to border on theism. For if mathematical truths have an independent existence—floating out there in the ether somewhere—then who created those truths? And by what means? Is it some

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higher power, with whom we would be well-advised to become better acquainted?

The Platonic view makes us more like physicists. It would not make much sense for a physicist to study his subject by simply making things up. Or cooking them up through pure cogitation. For the physicist is supposed to be describing the world around him. A physicist like Stephen Hawking, who is very creative and filled with imagination, is certainly capable of cooking up ideas like "black hole" and "supergravity" and "wormholes", but these are all intended to help explain how the universe works. They are not like manufacturing a fairy tale.

There are philosophical consequences for the thoughts expressed in the last paragraph. Physicists do not feel honor-bound to prove the claims made in their research papers. They frequently use other modes of discourse, ranging from description to analogy to experiment to calculation. If we mathematicians are Platonists, describing a world that is "already out there", then why cannot we use the same discourse that the physicists use? Why do we need to be so wedded to proofs?

One can hardly imagine an English Professor trying to decide whether his discipline is Platonic or Kantian. Nor would a physicist ever waste his time on such a quest. People in those disciplines know where the grist of their mill lives, and what they are about. The questions do not really make sense for them. We are somewhat alone in this quandary, and it is our job to take possession of it. If we can.

It appears that literary critics and physicists are certainly Platonists. What else could they be?[7] It is unimaginable that they would cook up their subject from within themselves. Certainly philosophers can and do engage in this discussion, and they would also be well-equipped (from a strictly intellectual perspective) to engage in the Platonic vs. Kantian debate. But they have other concerns. This does not seem to be their primary beat.

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[7]Although a physicist may put a finer point on it and assert that he has no care for a Platonic realm of ideas. Rather, he wishes to run experiments and "ask questions of nature."

The article [MAZ] sheds new and profound light on the questions being considered here. This is a discussion that will last a long time, and probably will never come to any clear resolution.

Once again the Platonic vs. Kantian debate illustrates the remove that mathematicians have from the ordinary current of social discourse. How can the layman identify with these questions? How can the layman even care about them? If I were a real estate salesman or a dental technician, what would these questions mean to me?

9   Seeking the Truth

In what we really understand, we reason but little.

— William Hazlitt

Mathematicians are good at solving problems. But we have recognized for a long time that we have a problem with communicating with laymen, with the public at large, with the press, and with government agencies. We have made little progress in solving this particular problem. What is the difficulty?

Part of the problem is that we are not well-motivated. It is not entirely clear what the rewards would be for solving this problem. But it is also not clear what the methodology should be. Standard mathematical argot will not turn the trick. Proceeding from definitions to axioms to theorems will, in this context, fall on deaf ears. We must learn a new *modus operandi*, and we must learn how to implement it.

This is not something that anyone is particularly good at, and we mathematicians have little practice in the matter. We have all concentrated our lives in learning how to communicate *with each other*. And such activity certainly has its own rewards. But it tends to make us blind to broader issues.

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It tends to make us not listen, and not perceive, and not process the information that we are given. Even when useful information trickles through, we are not sure what to do with it. It does not fit into the usual infrastructure of our ideas. We are not comfortable processing the data.

This is our own fault. This is how we have trained ourselves, and it is how we train our students. We are not by nature open and outreaching. We are rather parochial and closed. We are more comfortable sticking close to home. And, to repeat a tired adage, we pay a price for this isolation.

10   Brave New World

> For most wearers of white coats, philosophy is to science as pornography is to sex: it is cheaper, easier, and some people seem, bafflingly, to prefer it. Outside of psychology it plays almost no part in the functions of the research machine.
>
> — Steve Jones

For the past 2,000 years, mathematicians have enjoyed a sense of keeping to themselves, and playing their own tune.[8] It has given us the freedom to think our own thoughts and to pursue our own truths. By not being answerable to anyone except ourselves, we have been able to keep our subject pure and insulated from untoward influences.

But the world has changed around us. Because of the rise of computers, because of the infusion of engineering ideas into all aspects of life, because of the changing nature of research funding, we find ourselves not only isolated but actually cut off from many of the things that we need in order to prosper and grow.

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[8]Although it would be remiss not to note that Archimedes, Newton, and Gauss were public figures, and very much a part of society.

So it may be time to re-assess our goals, and our milieu, and indeed our very *lingua franca*, and think about how to fit in more naturally with the flow of life. Every medical student takes a course on medical ethics. Perhaps every mathematics graduate student should take a course on communication. This would include not only good language skills, but how to use electronic media, how to talk to people with varying (non-mathematical) backgrounds, how to seek the right level for a presentation, how to select a topic, and many of the other details that make for effective verbal and visual skills. Doing so would strengthen us as individuals, and it would strengthen our profession. We would be able to get along more effectively as members of the university, and also as members of society at large. Surely the benefits would outweigh the inconvenience and aggravation, and we would likely learn something from the process. But we must train ourselves (in some instances *re*-train ourselves) to be welcoming to new points of view, to new perspectives, to new value systems. These different value systems need not be perceived as inimical to our own. Rather they are complementary, and we can grow by internalizing them.

Mathematics is one of the oldest avenues of human intellectual endeavor and discourse. It has a long and glorious history, and in many ways it represents the best of what we as a species are capable of doing. We, the mathematics profession, are the vessels in which the subject lives. It is up to us to nurture it and to ensure that it grows and prospers. We can no longer do this in isolation. We must become part of the growing and diversifying process that is human development, and we must learn to communicate with all parts of our culture. It is in our best interest, and it is in everyone else's best interest as well.

REFERENCES

[**DAV**] E. B. Davies, Let Platonism die, *Newsletter of the European Mathematical Society* 64(1007), 24–25.

[**DYS**] F. Dyson, *Disturbing the Universe*, Basic Books, New York, 2001.

[**FER**] H. Ferguson, Sculpture Gallery,
`http://www.helasculpt.com/gallery/index.html`.

[**HOF**] D. Hoffman, The computer-aided discovery of new embedded minimal surfaces, *Math. Intelligencer* 9(1987), 8–21.

[**KRA**] S. Krantz, *The Proof is in the Pudding: A Look at the Changing Nature of Mathematical Proof*, Springer Publishing, to appear.

[**MAC**] S. Mac Lane, Mathematical models: a sketch for the philosophy of mathematics, *American Mathematical Monthly* 88(1981), 462–472.

[**MAN**] B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, New York, 1977.

[**MAZ**] B. Mazur, Mathematical Platonism and its opposites,
`http://www.math.harvard.edu/~mazur/`.

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