Practice Midterm 2

Note: These problems are more difficult than the ones you will encounter on the actual exam.

1. Suppose that $X$ is a random variable. Show that

$$P(X = 0) \leq \frac{V(X)}{E(X^2)}.$$

**Hint:** This can be proved by applying Cauchy’s inequality as follows: Let $Y = Y(X)$ be the random variable equal to 1 when $X \neq 0$, and let it equal 0 when $X = 0$. Then, it turns out that $X = XY$ (why?). Now, apply Cauchy’s inequality to both sides...

2. This problem was once asked to me by M. Halpin, and I think it is an excellent practice exercise: Suppose that $X$ is a uniformly distributed random variable over an interval $[-a, a]$. For what value $a$ does $X$ have mean 0 and variance 1?

3. Suppose that $X$ and $Y$ are independent standard normal random variables. Find the probability that $X + Y \leq 0$.

4. You conjecture that exactly 70% of the people in the U.S. prefer the color blue to the color green. You decide to test this hypothesis by taking a random sample of 1000 people. Suppose that you observe that only 500 people from this sample prefer the color blue over the color green. Let $P_{70}(X \leq 500)$ denote the probability that 500 or fewer people out of a sample of 1000 people, prefer blue to green, given that 70% of the whole population prefer blue to green. Assuming that our sample size is large enough so that $P_{70}$ can be approximated by a normal distribution via the Central Limit Theorem, calculate this probability $P_{70}(X \leq 500)$.

5. A certain math professor, with a class of 30 students, notices that in the first 5 minutes of class, only 15 students show up. Assuming that students arrival obeys a Poisson distribution over this 5 minute interval, calculate the probability that all 30 students show up in this first 5 minutes.

More to come...