1. Find \( dy/dx \):
   a. \( y^2 + y + \cos(x + y) = 0 \)
   b. \( \sin(x^2 + y^2) = 2x \).

2. Find the critical points, local minima, local maxima, absolute minima, absolute maxima, and points of inflection of the following function:

   \[
   f(x) = 4x^5 + 5x^4 + 1, \text{ where } x \in [-2, 3].
   \]

3. Use differentials (or the mean value theorem) to approximate
   a. \( 129^{2/7} \).
   b. \( \sin(\pi/3 + 1/20) \). (Hint: \( \sqrt{3}/2 = 0.8660254... \)).

4. A box with an open top has a square bottom, and has outer surface area equal to 100 square feet. Find the dimensions of the box that maximize its volume.

5. A certain planet’s orbit around a star is elliptical in a certain plane. After choosing a coordinate system appropriately, the star is assumed to be located at \((0, 0)\), and in this coordinate system, the star does not move. The location of the planet at time \( t \) in this coordinate system is given by \((2\cos(t), 3\sin(t))\). When the planet is closest to the star, it is said to be at perihelion, and when it is furthest from the star, it is said to be at aphelion. The planet will be at aphelion twice in a cycle (that is, twice a year), and will be at perihelion twice in a cycle. Let \( t_0 \) be any time at which the planet is at perihelion, and let \( t_1 \) be the next time after \( t_0 \) at which the planet is at aphelion. Let \( t_2 \) be the time midway between \( t_0 \) and \( t_1 \); that is, \( t_2 = \frac{t_0 + t_1}{2} \).

   How fast is the planet moving away from its star at this time \( t_2 \)?

   Hint: \( t_0 = 0 \) and \( t_1 = \pi/2 \).