Syllabus for Arithmetic Combinatorics

January 26, 2004

Instructor: Ernie Croot
Time and Place: The course meets MWF from 11:05-11:55 in room 108A Skiles.
Office: 125 Skiles
Office Hours: Tentatively, Mondays and Tuesdays from 1:15-2:30 or 3:00. I will usually be in my office in the afternoons most days, so if you can’t make these office hours, you might be able to catch me in.
Email: ecroot@math.gatech.edu
Webpage: www.math.gatech.edu/~ecroot Check this webpage for homework assignments, study notes, or additional information.
Homeworks: I plan to make homework assignments that are to be collected once every two weeks.
Exams: In a course like this, I don’t think it is a good idea to have standard, in-class exams, but I may give one for the final. Instead, I will give two special homework assignments which will count as midterms. For these assignments, you will be required to work by yourself (no collaboration with classmates), and without access to your notes or book.
Grading Policy: Your course grade will be based 30% on homework, 20% on each of the two midterms, and 30% on the final exam. In addition, I reserve the right to increase a person’s grade if he/she has demonstrated great talent and scholarship; for example, if you pull a Steve Smale (a Field’s medalist who is an emeritus professor at Berkeley, and who, as a graduate student, solved several famous problems in topology when he showed up for class one day, having skipped the previous lecture, saw a bunch of problems on the board, thought they were for homework, and then promptly handed in the solutions the next time the class met) then you will get an A for the course.
Text: I plan to use several texts for the course, two of which are the “Additive Number Theory:...” texts by Melvyn Nathanson. In my opinion,
Melvyn is one of the best expositors on Number Theory that there is, and it is well worth the time to read his books. Another book I plan to use is “The Hardy-Littlewood Method” by R. Vaughan. This is a fairly short book but is packed with an immense amount of material, which is typical of a British text, and is the style that I prefer. Finally, there are several great researchers in the field, such as W. T. Gowers and Ben Green, who have taken the time to write some fairly nice expository articles and to put them on their webpages, and I will use these articles for some of the lectures. Of course, I also plan to use some of my own notes and results for some of the lectures.

I strongly encourage you all to make the lectures. It is true that most of the course material can be found in a book, and perhaps you already know many of the results and proofs. However, you may not be so familiar with many of the underlying ideas, which I will give you in the lectures; and, understanding these ideas are essential if you want to do research in this area.

**Topic Matter:** I plan to cover the following topics in the order listed here:

1. Basic Ramsey Theory results: Schur’s theorem and van der Waerden’s Theorem. Perhaps I’ll mention generalizations, such as Rado’s theorem and the Hales-Jewitt theorem.
2. Basic inequalities: The Cauchy-Davenport theorem, Vosper’s theorem, the Erdos-Heilbron conjecture and Ruzsa’s “polynomial method” proof, Kneser’s inequality and applications, Shnirel’man’s addition theorem and applications.
3. Basic Structure Theory: Inverse theory of sumsets, special cases of Freiman’s theorem, the Frobenius problem, and “Nathanson’s theorem”.
4. Elementary methods for counting points on varieties involving Gauss and Jacobi sums, the method of exponential sums, Weyl’s equidistribution theorem, bounds for exponential sums, Waring’s problem, the large sieve and applications.
5. Basic harmonic analysis, including Roth’s theorem and Szemeredi and Heath-Brown’s improvement, as well as Sarkozy’s result on differences in a set equal to a square.
6. Deeper combinatorial methods: the Plunnecke inequalities, Ruzsa’s proof of Freiman’s theorem for finite abelian groups, the Szemeredi regularity lemma and applications, Freiman’s theorem for the integers.
7. Finally, I may lecture on some really big recent theorems, such as Gowers’s “effective Szemeredi” theorem, or maybe some really beautiful results due to Ben Green.