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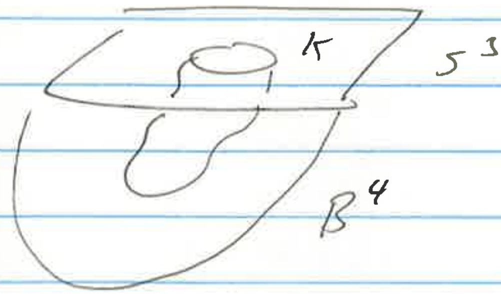
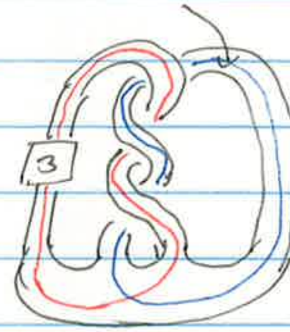
TTC 2013

Counterexamples to Conjectures of Kauffman on Slice Knots
(joint with Christopher Davis)

Slice Knot K



$K = \partial F$ $\quad F =$ Seifert surface in S^3



$\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$

Def^{n}: Seifert Matrix V

choose a basis of $H_1(F) \cong \mathbb{Z}^{2g} = \{a_i \mid i = 1, \ldots 2g\}$

$$V_{ij} = lk(a_i, a_j^{+})$$

<u>Prop:</u> if K is slice knot then for <u>any</u> Seifert surface F there is a basis st.

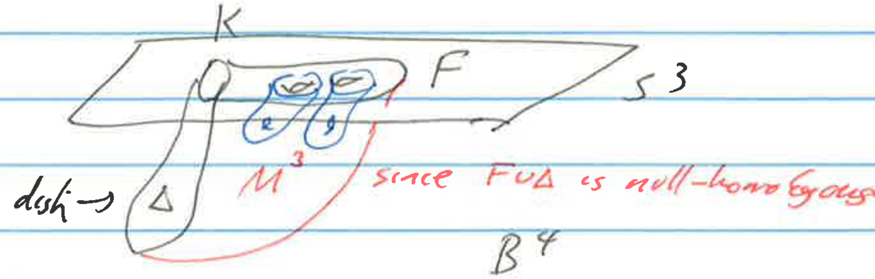
$$V = \begin{pmatrix} \underset{g\times g}{0} & * \\ * & * \end{pmatrix}$$

$\Leftrightarrow \exists$ link $\{d_1 - d_g\}$ st. $lk(d_i, d_j) = 0$

K satisfies conclusion is <u>algebraically slice knot</u> and $\{d_1 \ldots d_g\}$ a <u>derivative</u> of K

Slice $\Rightarrow$ Alg. slice $\Leftrightarrow$ Any Seif. surface admits a derivative link

Proof:



½ lives ½ dies lemma:

$\ker H_1(F;\mathbb{Q}) \to H_1(M)$ is $\cong \mathbb{Q}_g$

the curves that "die" give derivative link

If M^3 could be taken to be a solid handlebody then $\exists$ derivative link of F that is itself a slice link

classical obstr. invts from V

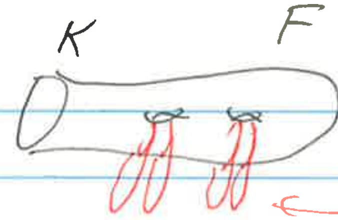
signatures: $|\omega| = 1 \quad \omega \in \mathbb{C}$

$\sigma_\omega(K) = \text{signature}((1-\omega)V + (1-\bar{\omega})V^T)$

get signature function on S^1

Prop 2: if K has a derivative that is slice then K is slice

Proof:



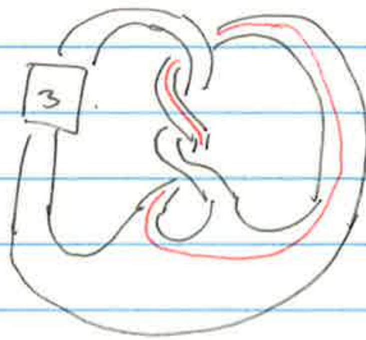
(compress F on disks)

← got disk

Levine: $S^{2n-1} \hookrightarrow S^{2n+1}$ $n > 1$

then slice $\Leftrightarrow$ alg slice

find derivative of genus 1:



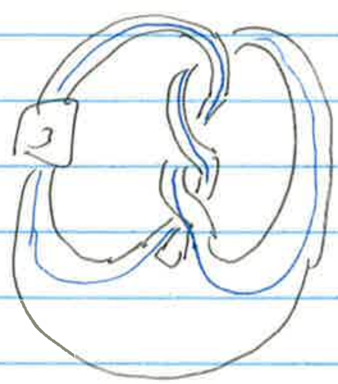
$$(x\ y)\begin{pmatrix}3 & 2\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = 0$$

$$3x^2 + 2xy + xy = 0$$

$$3x^2 + 3xy = 0$$

$$x = 0$$

$$y = -x$$



Louis Kauffman Conjectures: K genus 1

K $\Rightarrow$ on any Seif. surface one of the derivatives is a slice knot

$\Rightarrow$ for one of the derivatives $\sigma_\omega = 0$ $\forall \omega$

$\Rightarrow$ for " " Arf $= 0$

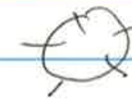
untwisted Whitehead double of K

Special case

Wh(J)

Conjecture: Wh(J) is slice $\Longleftrightarrow$ J is slice

Evidence: 1) K slice $\Rightarrow$ one of the derivatives J has $\sum \sigma_{\omega_i}(J) = 0$ certain ω_i's in



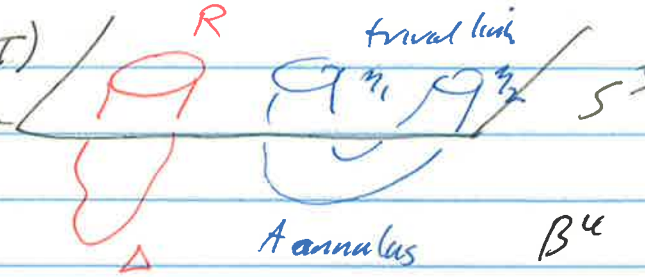
2) higher order (n^{th}) ℓ^2-signatures of K vanish $\Rightarrow$ $(n-1)$ order ℓ^2-signatures of J vanish

3) Wh(J) is slice $\Rightarrow$ $\tau(J) \geq 0$

Th^m A: There exists sliceknots with unique genus one Seifert surface on which each derivative has non-zero Arf and signatures

("counter examples" to slice whitehead conjecture in homology 3-spheres)

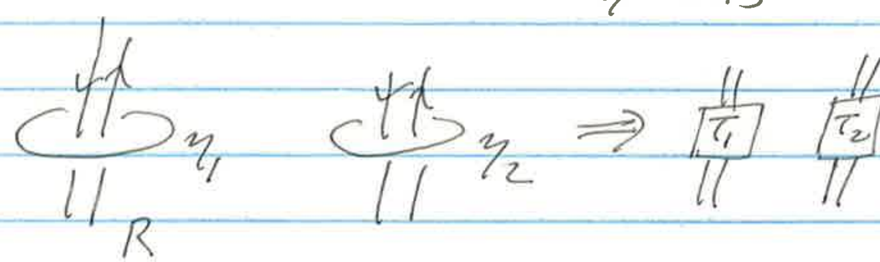
$Th^{\underline{m}}$: The knot $R' = R(\eta_1, \eta_2, T)$ obtained by cutting out $A \times D^2$ and putting in $E(T) \times [0,1]$ is slice



easy to see result is a homology B^4 & $\pi_1 = 0$ so homeo to B^4 can show diffeo to B^4 too.

<u>Method of Altering knots in S^3:</u> Infection

Suppose $\{\eta_1, \eta_2\}$ is trivial link in S^3 and $lk(\eta_1, R) = 0$ and T_1, T_2 any knots



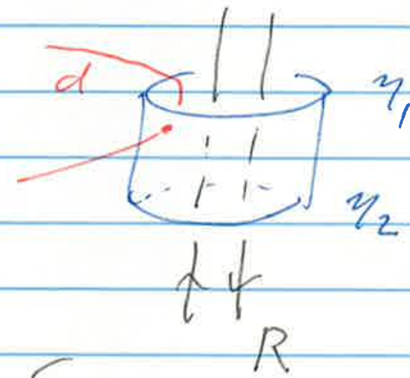
(same as: cut out $\eta_1 \times D^2$ and glue in $E(T_i)$)

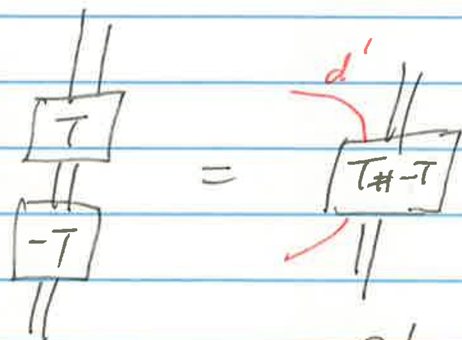
Alter $(R, F, d = \text{slice knot})$ to $(R', F', d'$ has nonzero signature)
slice — derivative
R' slice

Suppose: 1) η_1, η_2 form trivial link in $S^3 \setminus F$

2) η_1, η_2 cobound annulus A in $S^3 \setminus R$

3) $d \cap A = 1$ point





= slice knot

$R' = R(\eta_1, \eta_2, T, -T)$

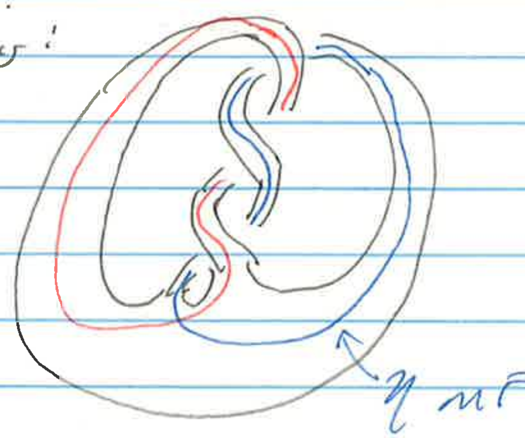
with (F', d')

$d \rightsquigarrow d' = d \# T$

How can we achieve this:

let $\eta_1 = \eta^+$

$\eta_2 = \eta^-$



takes a little work to make this work to get counterexample