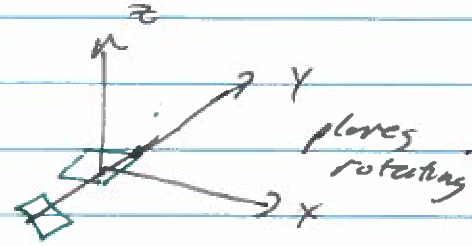
Caitlin Leverson
TTC '16

Invariants of Legendrian Knots

$\mathbb{R}^3$, $\#^k(S^1 \times S^2)$ ambient space

def^{n}: the standard contact structure on $\mathbb{R}^3$ is the 2-plane field $\xi = \ker \alpha$

$$\alpha = dz - y\,dx$$



a Legendrian knot is

$$\Lambda : S^1 \to (\mathbb{R}^3, \xi)$$

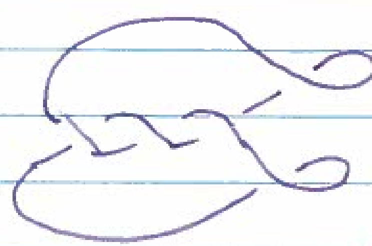
in such a way that Λ is everywhere tangent to the contact planes

two projections:

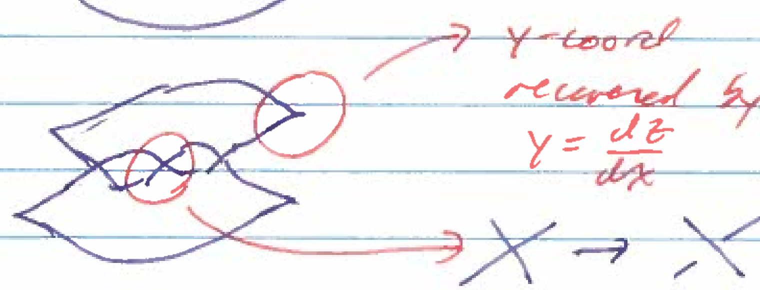
- the Lagrangian projection $(x,y,z) \longmapsto (x,y)$
- the front projection $(x,y,z) \longmapsto (x,z)$

example Right handed trefoil

Lagrangian



Front



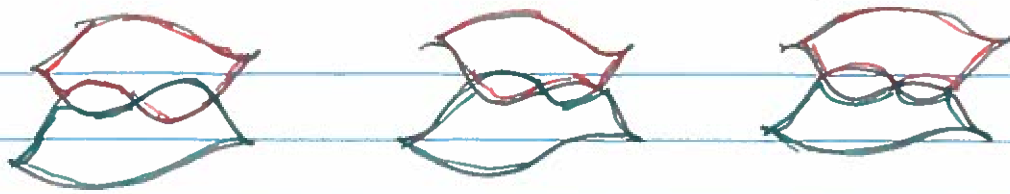
(1) <u>Chekanov - Eliashberg DGA</u>
over $\mathbb{Z}[t, t^{-1}]$
(Etnyre - Ng - Sabloff)

- generated by crossings in Lagrangian projection
- grading (ignore for this talk)
- differential determined by count of immersed polygons in Lagrangian proj.

$$(\mathcal{A}(\Lambda), \partial)$$

Def: let F be a field.
an <u>augmentation</u> of $(\mathcal{A}, \partial)$ to F is a ring map $\varepsilon : \mathcal{A} \to F$ such that
$\varepsilon \circ \partial = 0$ and
$\varepsilon(1) = 1$

(2) <u>normal rulings</u>
front diagram has n right cusps
decompose front projection into n circles
- each circle bounds a topological disk
- 1 left, 1 right cusp
- intersections only at crossings in particular ways



want to relate rulings and augmentations

Fuchs, Fuchs-Ishkhanov, Sabloff

$\underline{\text{Th}^{\text{m}}}$: let Λ be a Legendrian knot

$$\left\{\begin{matrix}\text{normal}\\ \text{ruling}\end{matrix}\right\} \underset{\text{many to one}}{\rightleftarrows} \left\{\begin{matrix}\text{aug to } \mathbb{Z}/2\\ \text{of } (\mathcal{A}_{\mathbb{Z}/2}, \partial)\end{matrix}\right\}$$

what about with $\mathbb{Z}[t,t^{-1}]$ coeff.

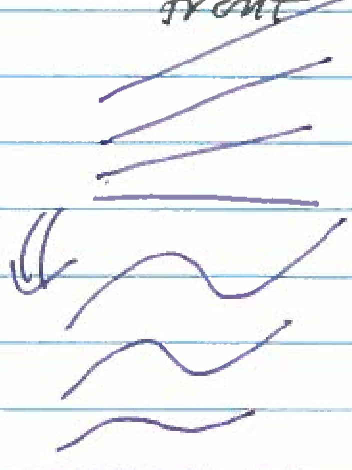
$\underline{\text{Th}^{\text{m}}(L)}$:

let Λ be a Leg. knot

$$\left\{\begin{matrix}\text{normal}\\ \text{ruling}\end{matrix}\right\} \rightleftarrows \left\{\begin{matrix}\text{aug to } F\\ \text{of } (\mathcal{A}_{\mathbb{Z}[t,t^{-1}]}, \partial)\end{matrix}\right\}$$

<u>Outline of Proof:</u>

front

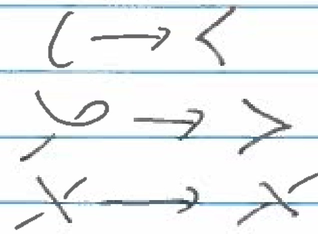


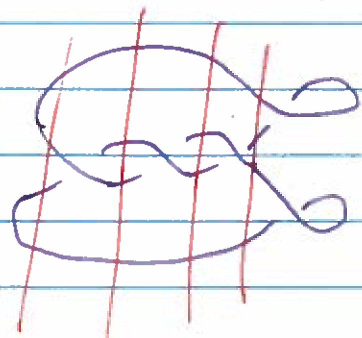
Lag

this "localizes" differential

(aug ⟶ ruling)

aug of original Lag ⟶ aug of dipped diag (i.e. do [illegible] more) ⟶ get front diagram





do dip more

augmentation gives crossing changes of ruling

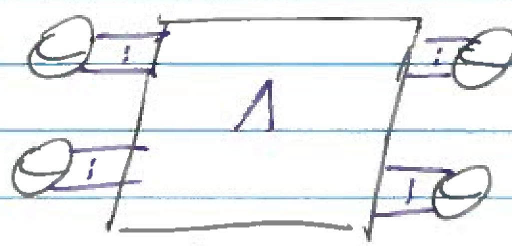
(ruling ⟶ aug)

normal ruling of front diag ⟶ aug. of dipped diag ⟶ aug of Lag. ✓

—— // ——

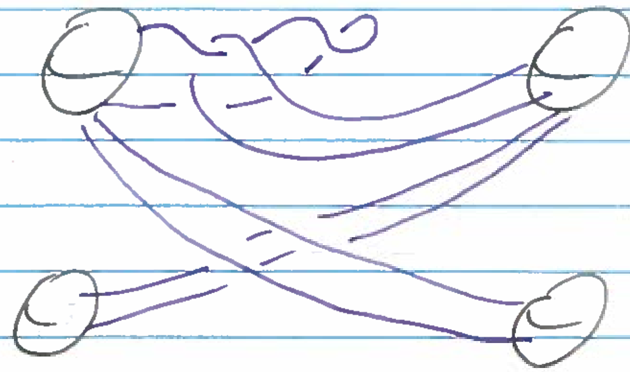
now Legendrians in $\#^n(S^1 \times S^2)$

by Gompf these are

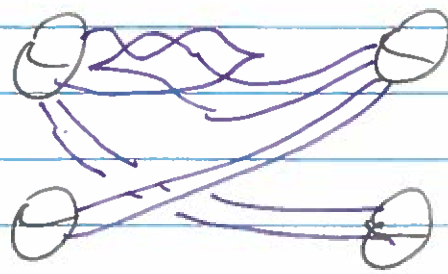


tangle

Lag projection



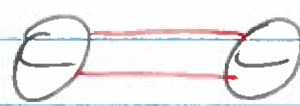
Front projection



Two invariants:

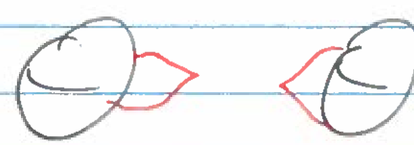
1. Chekanov-Eliashberg DGA over $\mathbb{Z}[t, t^{-1}]$ (Ekholm-Ng)
2. Extended def^{n} of normal rulings (L.)

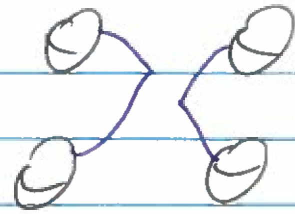
"Def^{n}"

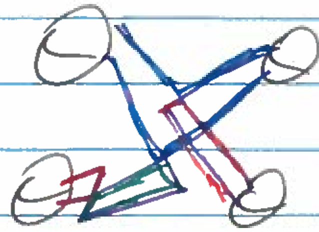
1) 

2) 

no way to "pair strands" so no normal rulings

3) 

4)  paired strands must go through some handle so no normal ruling

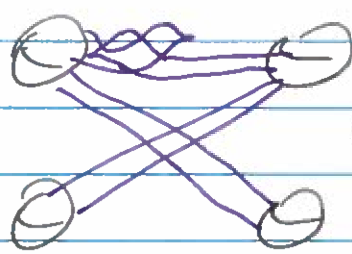
5) 

Thm (L.): same idea in $\mathbb{R}^3$
$\{\text{aug } s\} \longleftrightarrow \{\text{rulings}\}$

idea of proof:

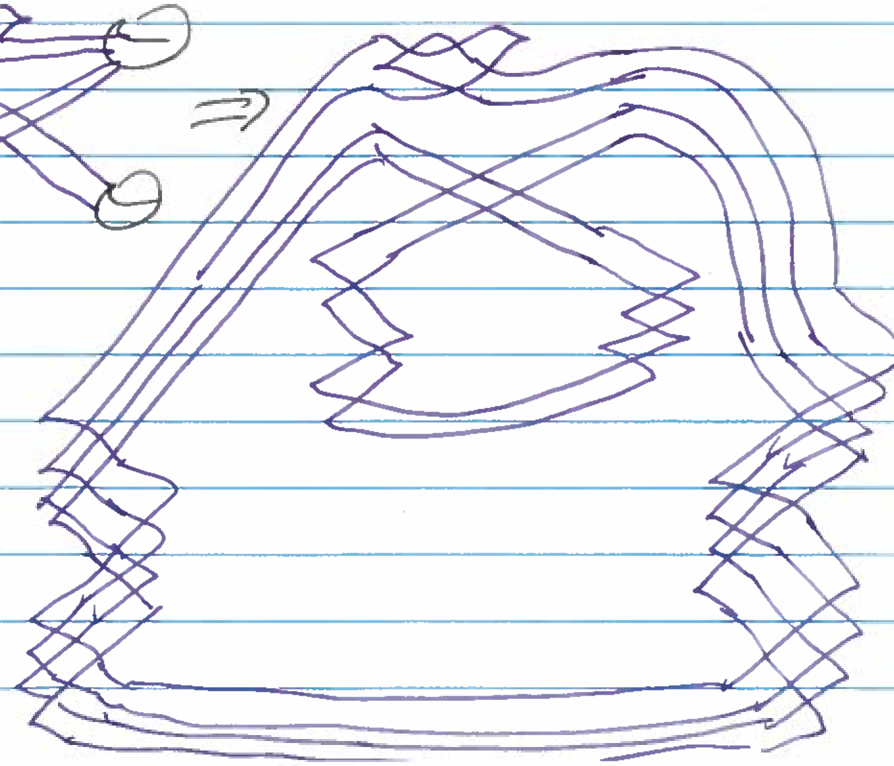
$\Lambda \subseteq \#^k(S^1 \times S^2)$

consider $S(\Lambda) \subset \mathbb{R}^3$



$\Rightarrow$

$S(\Lambda)$



(aug $\to$ ruling)

aug of $\Lambda \subset \#^h S^1 \times S^2$

$\downarrow$

aug of $S(\Lambda) \subset \mathbb{R}^3$

$\downarrow$

normal ruling $S(\Lambda) \subset \mathbb{R}^3$

$\downarrow$

gives ruling of $\Lambda \subset \#^h S^1 \times S^2$

(ruling $\to$ aug) similar idea ✓

why care about $\Lambda \subset \#_h (S^1 \times S^2)$

any Weinstein mfd W is 2-handle attachment to Legendrian link in $\#_h (S^1 \times D^3)$

$\Lambda \subset \#^h (S^1 \times S^2)$

<u>thm (BEE)</u>: if Λ has augmentation to $\mathbb{Q}$, then Symplectic Homology$(W) \neq 0$

previous thm says rulings (easier) $\Rightarrow SH(W) \neq 0$