Lightning Talks I
Tech Topology Conference

December 7, 2018

Concordance of knots in 3-manifolds

JungHwan Park
(joint with Matthias Nagel, Patrick Orson, and Mark Powell)

Georgia Institute of Technology

Tech Topology Conference

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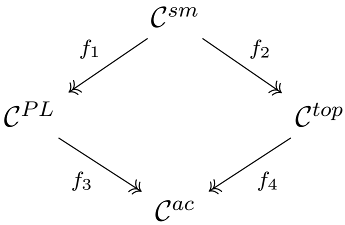
Knots $K_0, K_1 \subset S^3$ are *almost concordant* if $K_0 \# J$ is topologically concordant to K_1 for some $J \subset S^3$.

Natural maps



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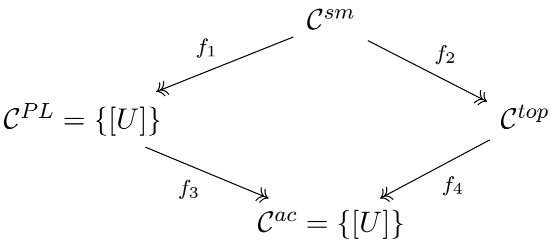

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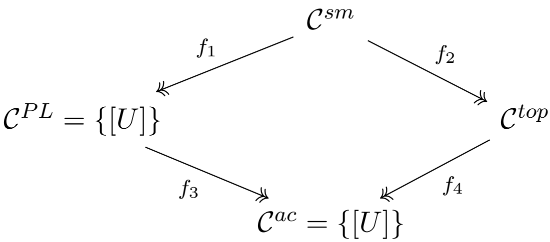


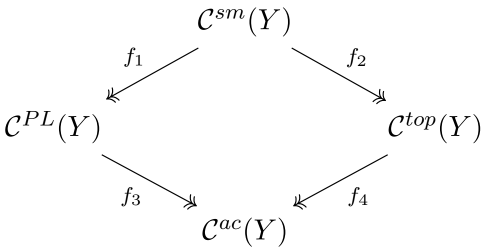
Theorem (Hom (2015), Ozsváth-Stipsicz-Szabó (2017))

$\ker f_2 \cong \mathbb{Z}^\infty \oplus G'$.

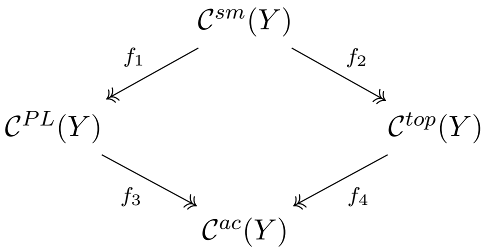
Theorem (Hedden-S. Kim-Livingston (2016))

$\ker f_2 \geq \mathbb{Z}_2^\infty$.

Natural maps



Natural maps



Theorem (Friedl-Nagel-Orson-Powell (2018))

For $Y \neq S^3$, $|\mathcal{C}^{ac}(Y)| = \infty$.

Natural maps

$$\begin{array}{ccccc} & & \mathcal{C}^{sm}(Y) & & \\ & \overset{f_1}{\swarrow} & & \overset{f_2}{\searrow} & \\ \mathcal{C}^{PL}(Y) & & & & \mathcal{C}^{top}(Y) \\ & \underset{f_3}{\searrow} & & \underset{f_4}{\swarrow} & \\ & & \mathcal{C}^{ac}(Y) & & \end{array}$$

Theorem (Friedl-Nagel-Orson-Powell (2018))

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Theorem (Nagel-Orson-P.-Powell (2018))

For $Y \neq S^3$, $|f_i^{-1}([U])| = \infty$, *for* $i = 1, 2, 3, 4$.

Length spectra of q-differential metrics

Marissa Loving
University of Illinois at Urbana-Champaign

A few warm-up questions…

How many measurements to determine a square?

How many measurements to determine a square?

How about a rectangle?

How about a rectangle?

Or a parallelogram?

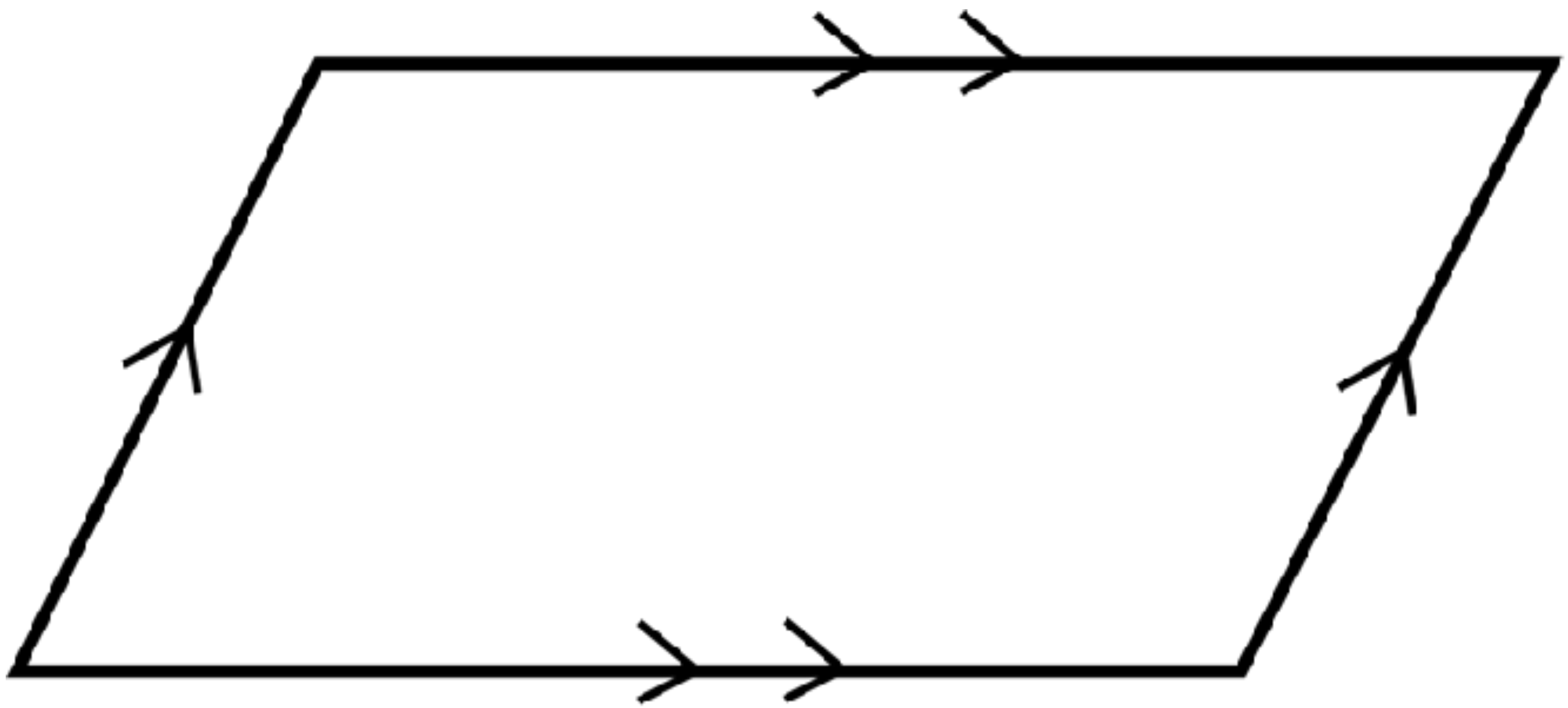
Or a parallelogram?

What if we consider surfaces instead?

How many curves' lengths do we need to know to determine a surface?

For example, lets consider a flat torus.

Flat torus:

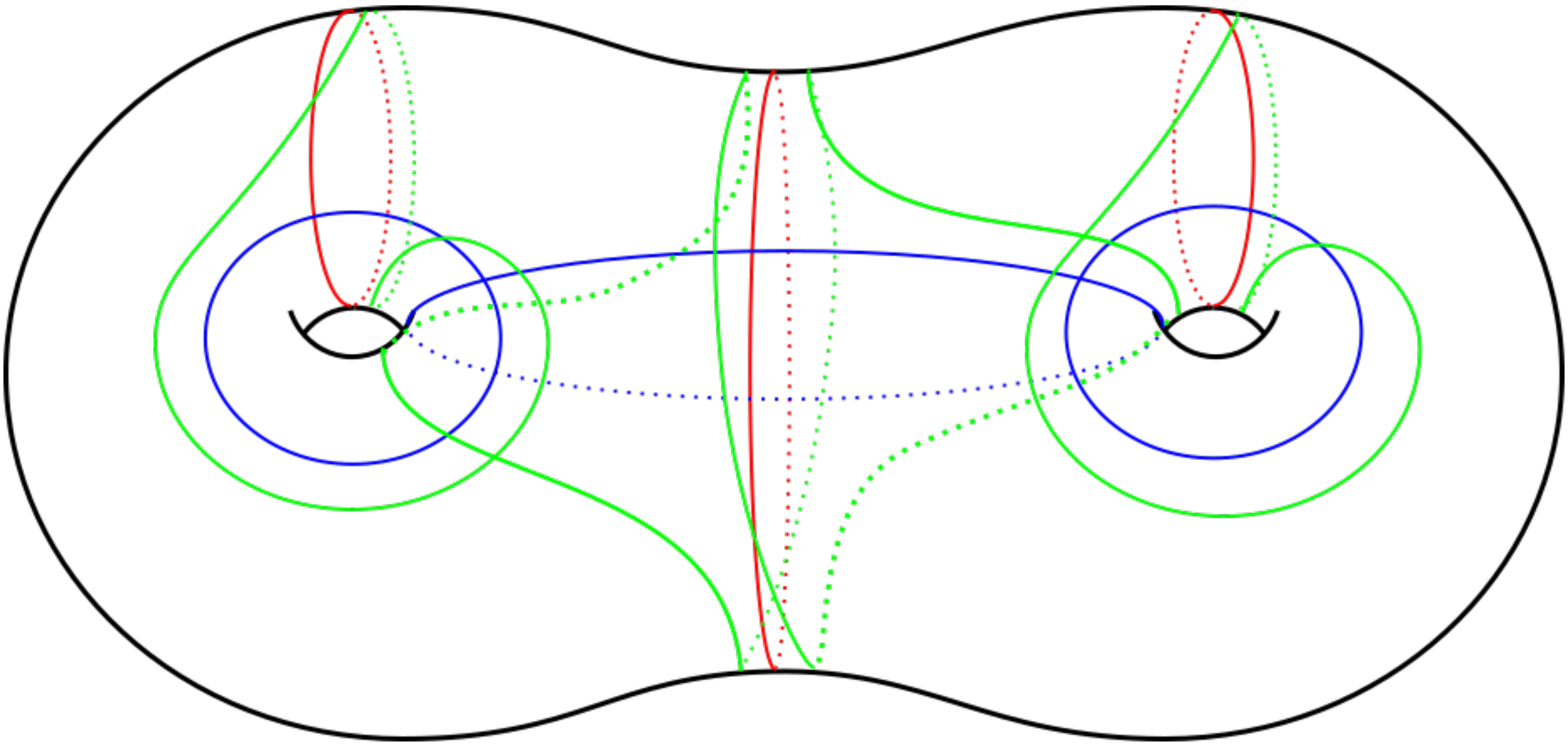


How many curves do we need to determine a flat metric on the torus?

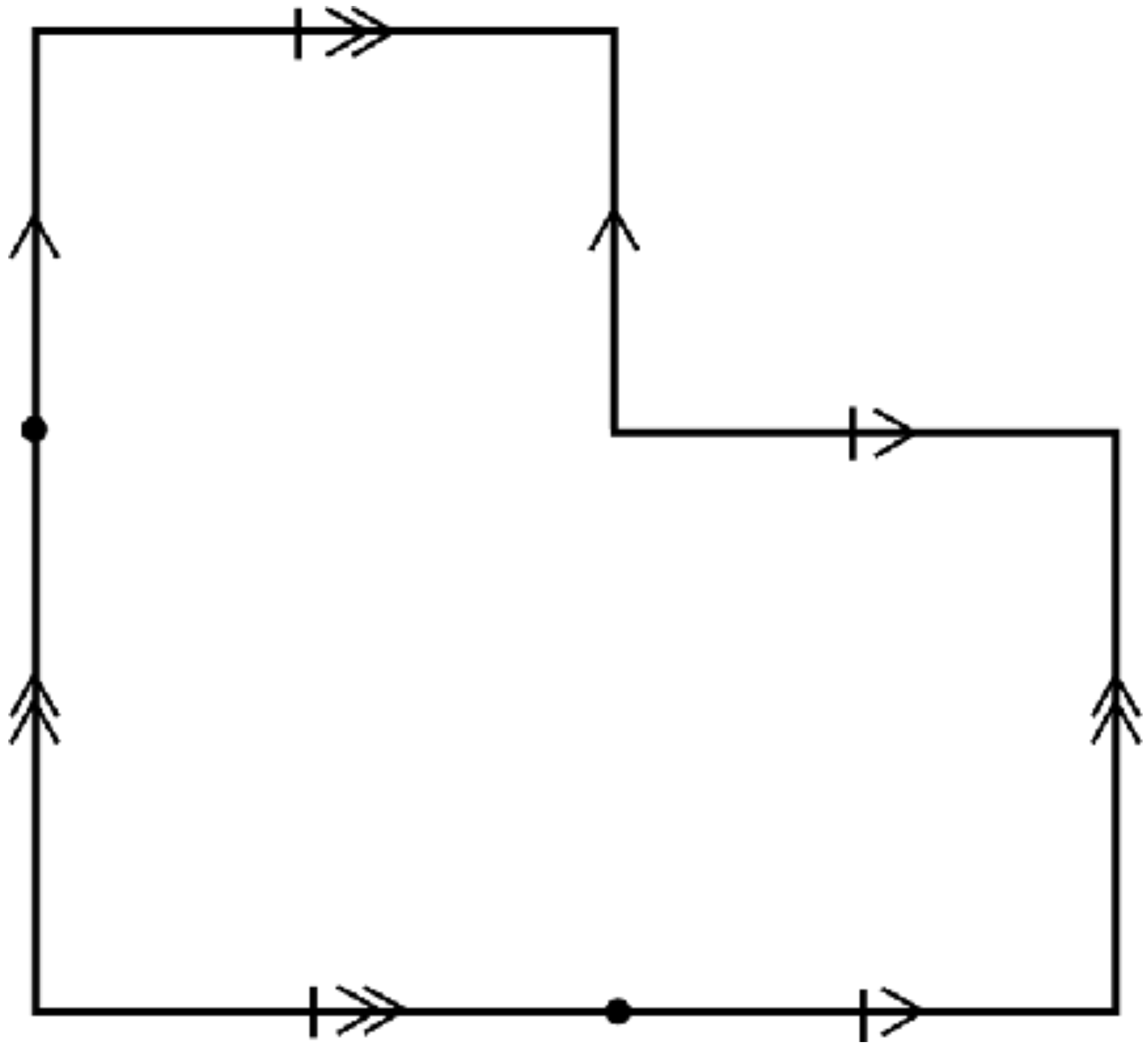
How many curves are needed to determine a hyperbolic metric on a closed surface of genus g?

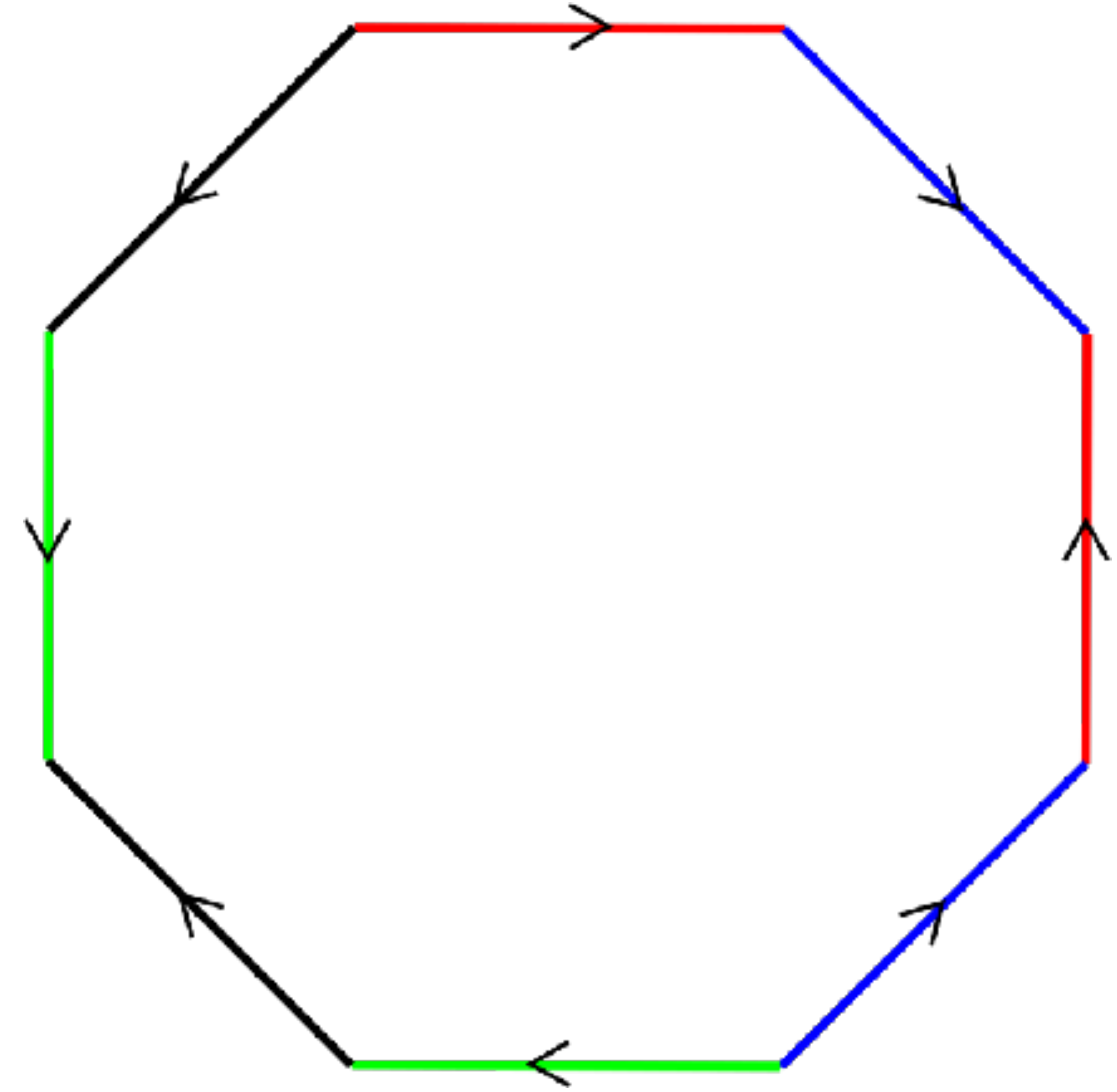
The 9g-9 Theorem

For g = 2, we need 9 curves.



How many curves are needed to determine a flat metric on a closed surface?



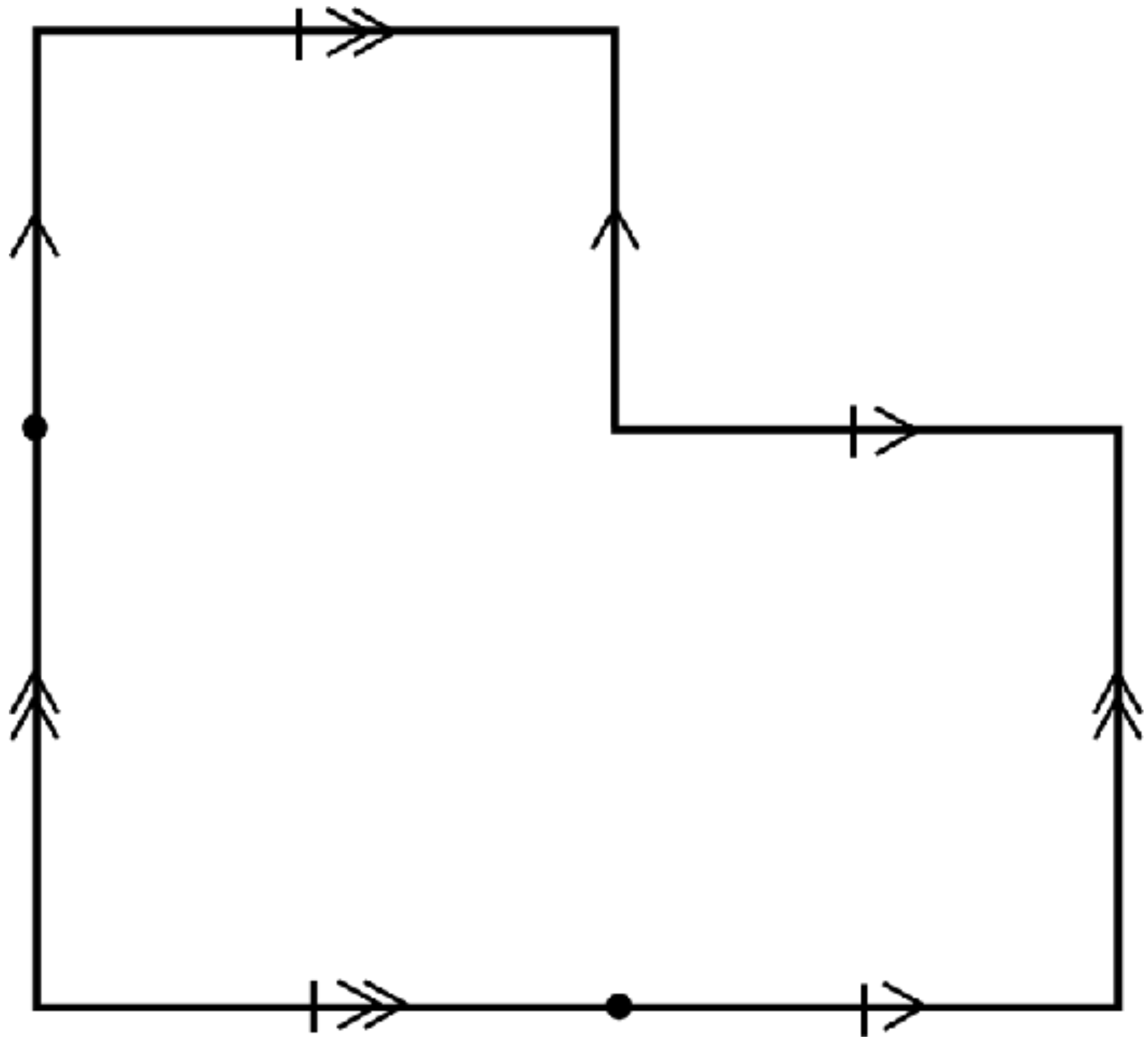


Thm. (Bankovic—Leininger)

To determine an arbitrary flat metric on a closed surface you need the lengths of **ALL** closed curves.

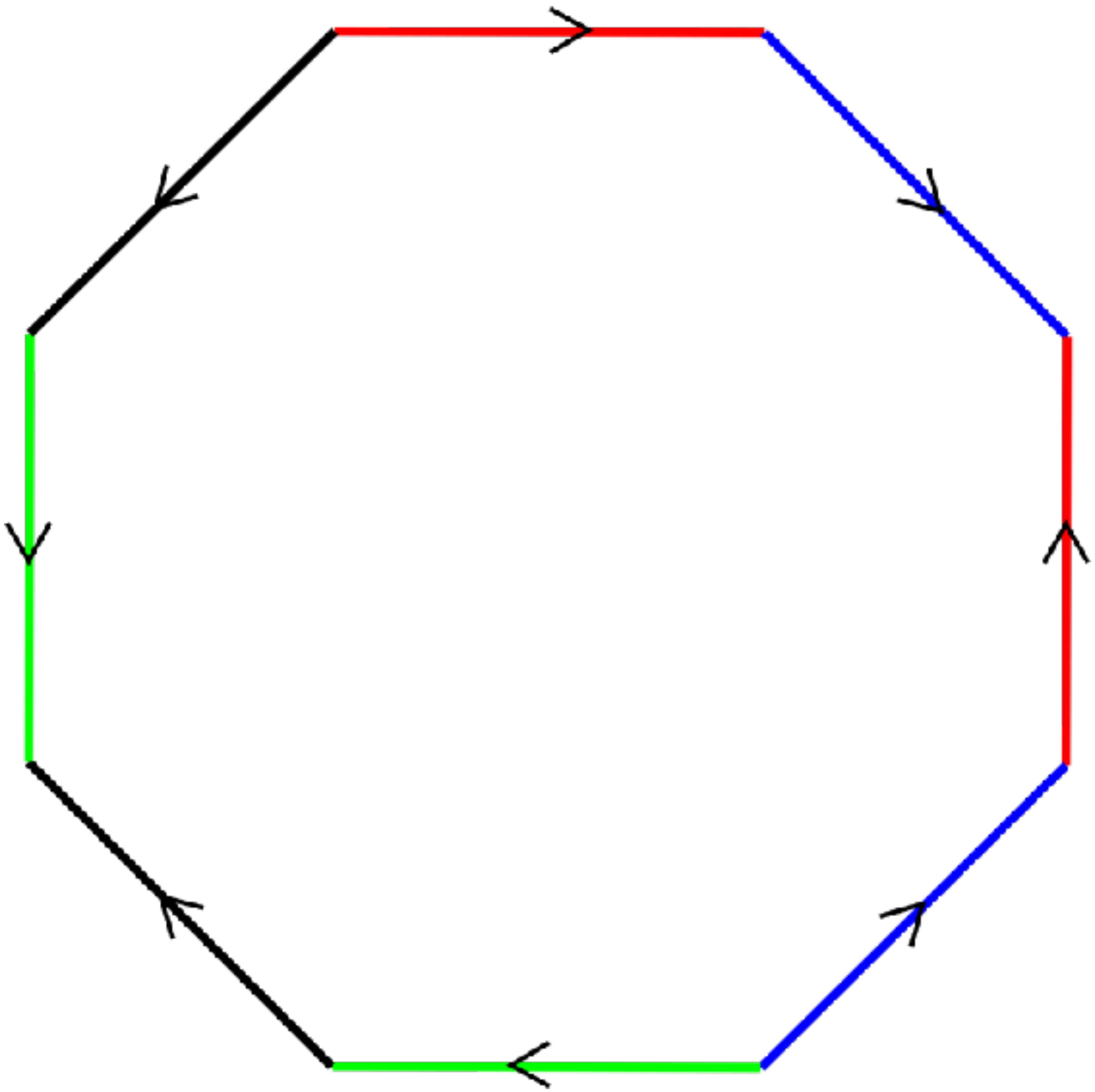
Thm. (Duchin—Leininger—Rafi)

To determine a flat metric coming from a quadratic differential you only need the lengths of **simple** closed curves.



Thm. (Loving)

To determine a flat metric coming from a q-differential you only need the lengths of **q-simple** curves.



Thank you!!!

An infinite rank summand of the homology cobordism group

Linh Truong
(joint work with I. Dai, J. Hom, and M. Stoffregen)

Columbia University

Tech Topology Conference, December 2018

The homology cobordism group

Definition

Y_1 and Y_2 are **homology cobordant**, denoted $Y_1 \sim Y_2$, if they cobound a smooth, compact, oriented cobordism W such that the inclusions $H_*(Y_i;\mathbb{Z}) \to H_*(W;\mathbb{Z})$ induce isomorphisms on homology.

Definition

The homology cobordism group $\Theta^3_{\mathbb{Z}}$ is defined as

$$\Theta^3_{\mathbb{Z}} = \{\text{oriented integral homology three-spheres}, \#\}/\sim$$

Background + Main Theorem

Theorem (Finteshel-Stern, '85)

The group $\Theta^3_{\mathbb{Z}}$ is infinite.

Theorem (Furuta '90, Finteshel-Stern '90)

The group $\Theta^3_{\mathbb{Z}}$ contains a $\mathbb{Z}^\infty$ subgroup.

Theorem (Dai, Hom, Stoffregen, T. '18)

The group $\Theta^3_{\mathbb{Z}}$ contains a $\mathbb{Z}^\infty$ summand.

Ingredients in the proof

We build on the **Involutive Heegaard Floer homology** package of Hendricks-Manolescu and Hendricks-Manolescu-Zemke. We define an **almost local equivalence group** $\widehat{\mathfrak{I}}$ and consider a homomorphism

$$\widehat{h} : \Theta^3_{\mathbb{Z}} \to \widehat{\mathfrak{I}}$$

which factors through the Hendricks-Manolescu-Zemke homomorphism $h : \Theta^3_{\mathbb{Z}} \to \mathfrak{I}$.

Inspired by work of Hom on the knot concordance group, we prove:

Theorem (Dai, Hom, Stoffregen, T.)

The almost local equivalence group $\widehat{\mathfrak{I}}$ is totally ordered.

Ingredients in the proof (continued)

We prove a classification theorem for $\widehat{\mathfrak{I}}$, which leads to:

Theorem (Dai, Hom, Stoffregen, T.)

For every $n \in \mathbb{N}$ there are surjective homomorphisms $\phi_n : \widehat{\mathfrak{I}} \to \mathbb{Z}$.

The Brieskorn spheres $Y_i = \Sigma(2i+1, 4i+1, 4i+3)$ satisfy $\phi_j \circ \widehat{h}(Y_i) = \delta_{ij}$. Hence,

$$\{\phi_n \circ \widehat{h}\}_{n \in \mathbb{N}} : \Theta^3_{\mathbb{Z}} \to \mathbb{Z}^\infty$$

is a surjective homomorphism.

Open questions

1. Does there exist any torsion in $\Theta^3_{\mathbb{Z}}$?
2. Is $\Theta^3_{\mathbb{Z}}$ generated by Seifert fibered spaces?
3. Is every element in $\Theta^3_{\mathbb{Z}}$ represented by Dehn surgery on a knot?

Thanks for listening!

Enriching Bézout's Theorem

Stephen McKean (Georgia Tech)

December 7th, 2018

Tech Topology Conference 2018

Bézout's Theorem

Theorem

Let k be an algebraically closed field. If $f, g \subset \mathbb{P}^2_k$ are generic algebraic curves of degree c, d, respectively, then

$$\sum_{p \in f \cap g} i_p(f, g) = cd.$$

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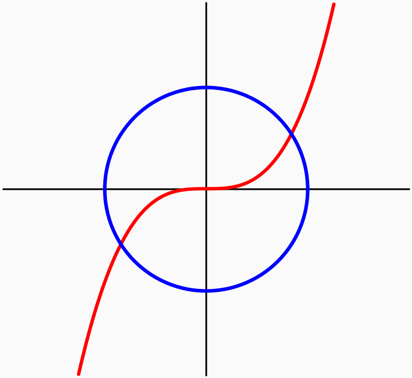

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- Use $\deg^{\mathbb{A}^1}$ to enrich enumerative results in quadratic forms. (Kass-Wickelgren, et al.)
- Enriched results carry extra information.

Enriched Bézout's Theorem

Theorem (McKean)

Let k be a perfect field and f, g be transverse of degrees c, d with $c + d$ odd.

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- Over $\mathbb{C}$: counts intersection points.
- Over $\mathbb{R}$: equal number of positive/negative crossings.
- Over $\mathbb{F}_q$: counts crossing types mod 2.

Example

$$k = \mathbb{R}, \quad f = y - x^3, \quad g = y^2 + x^2 - 1.$$

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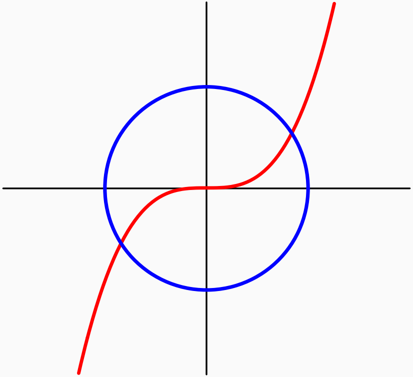

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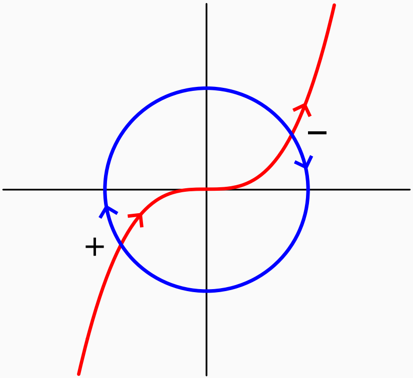


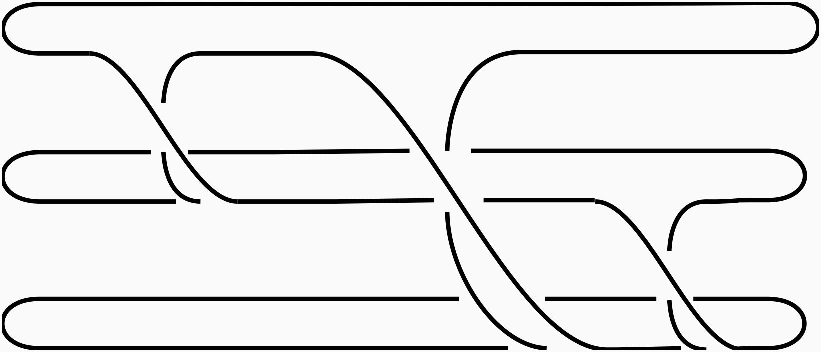
Quasipositive Surfaces and Convex Surface Theory

Moses Koppendrayer

12/7/18

University of Miami

Quasipositive surfaces were originally defined by Lee Rudolph to be the standard Seifert Surface of a strongly quasipositive braid.



Motivating Question

If a knot has a quasipositive surface, must every minimal genus seifert surface be quasipositive?

Baader and Ishikawa showed in S^3, quasipositive surface $\Longleftrightarrow$ isotopic to the ribbon of a Legendrian graph.

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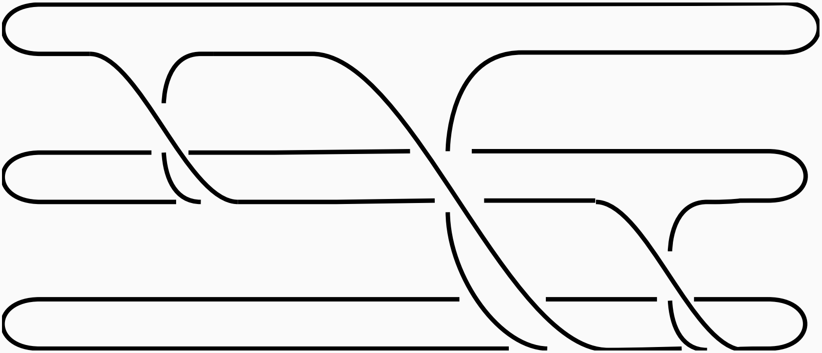

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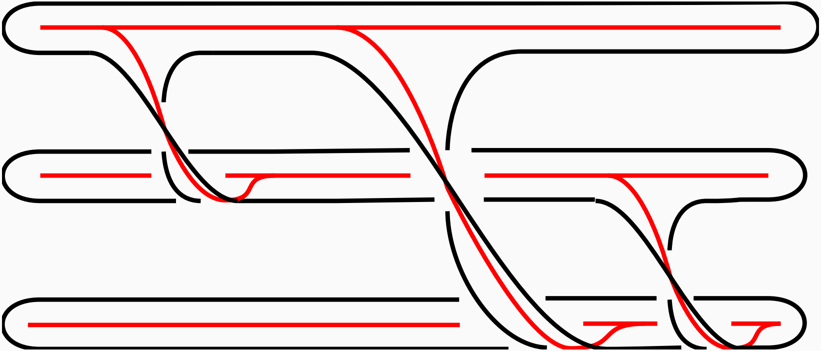

What are the advantages of this viewpoint?

- Additional structure relating to the ambient manifold allows us to use tools from contact topology.

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- Legendrian ribbonness is a property that you can consider in any contact manifold.

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- Additional structure relating to the ambient manifold allows us to use tools from contact topology.
- Legendrian ribbonness is a property that you can consider in any contact manifold.
- It allows us to 'refine' the notion of quasipositivity. I.e. the difference between being topologically isotopic to a Legendrian ribbon and being presented as a Legendrian ribbon.

Theorem[In progress]

There exists a transverse link in $L(4,1)$ with two interior disjoint Seifert surfaces, R and S such that:

1) R is the ribbon of a Legendrian graph

2) S is topologically isotopic to the ribbon of a Legendrian graph

3) Any isotopy of S to the ribbon of a Legendrian graph doesn’t restrict to a transverse isotopy of the boundary.

Sebastian Baader and Masaharu Ishikawa.
Legendrian graphs and quasipositive diagrams.
Ann. Fac. Sci. Toulouse Math. (6), 18(2):285–305, 2009.

Vincent Colin.
Chirurgies d'indice un et isotopies de sphères dans les variétés de contact tendues.
C. R. Acad. Sci. Paris Sér. I Math., 324(6):659–663, 1997.

Ko Honda.
On the classification of tight contact structures. I.
Geom. Topol., 4:309–368, 2000.

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Constructions of quasipositive knots and links. I.
In *Knots, braids and singularities (Plans-sur-Bex, 1982)*, volume 31 of *Monogr. Enseign. Math.*, pages 233–245. Enseignement Math., Geneva, 1983.

The Geometry of the Separating Curve Graph

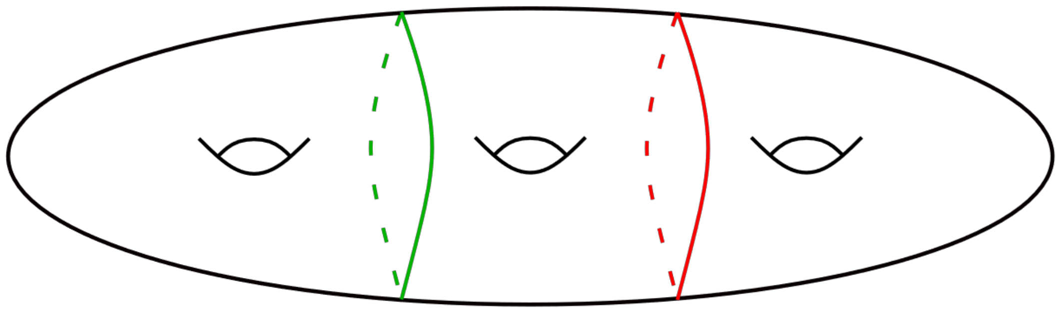
Jacob Russell

THE GRADUATE CENTER
THE CITY UNIVERSITY OF NEW YORK

The Separating Curve Graph

Vertices: Separating curves on S

$\mathrm{Sep}(S)$ 



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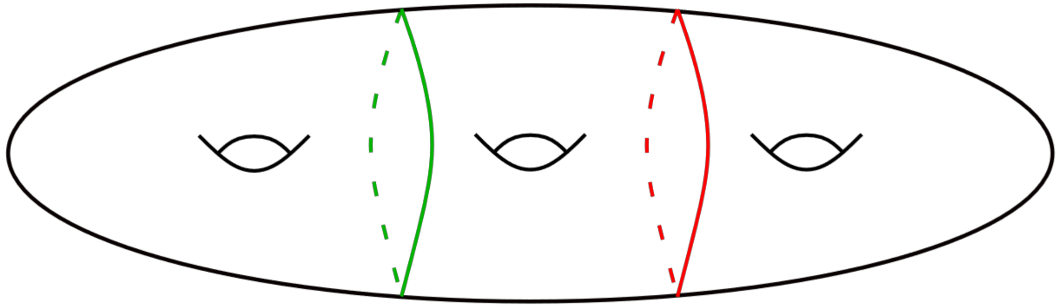

Edges: Disjointness

The Separating Curve Graph

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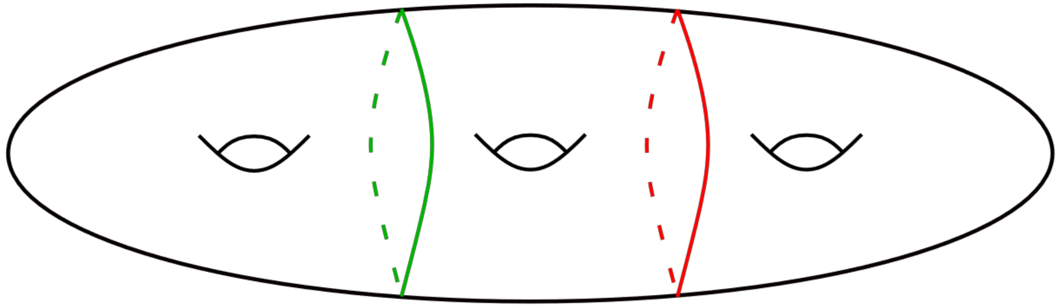
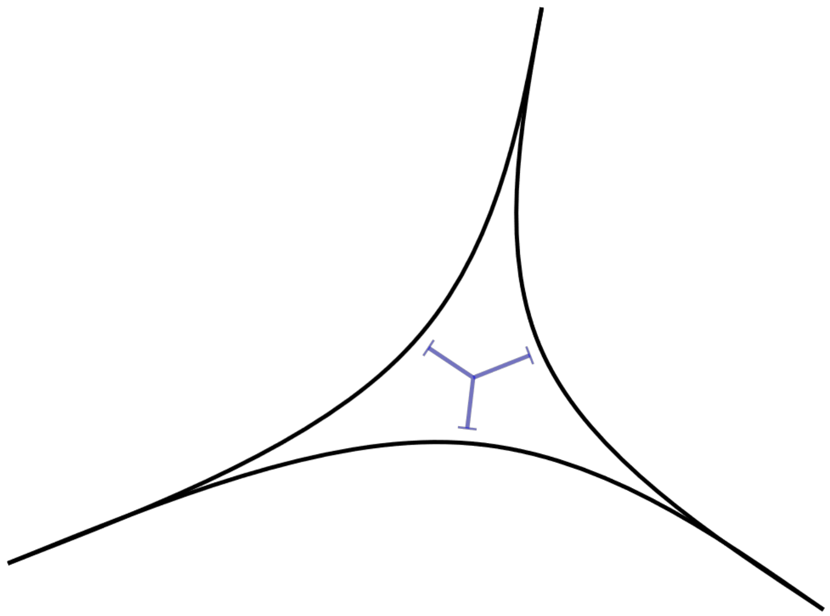


Edges: Disjointness

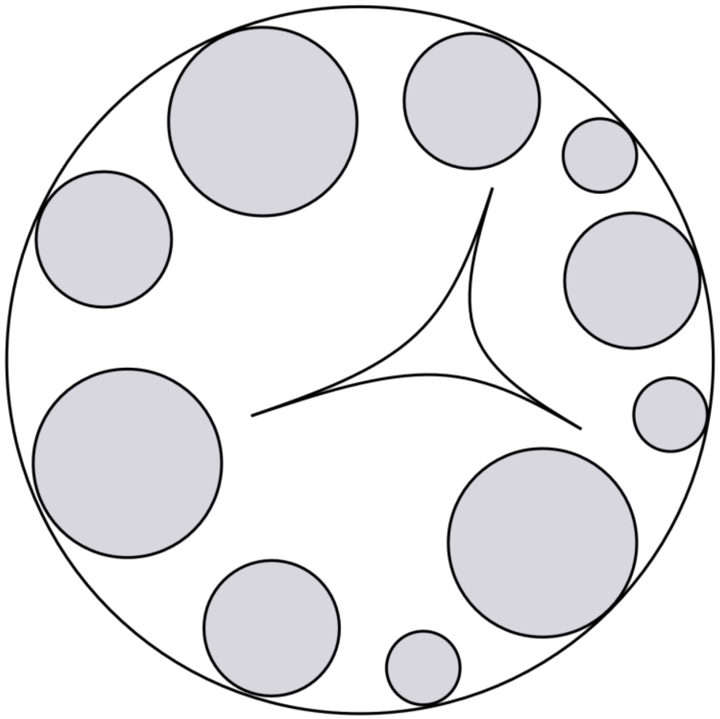
Goal: Study large scale geometry of $\mathrm{Sep}(S)$

Theorem (Vokes) $\mathrm{Sep}(S)$ is hyperbolic if and only if S has at least 3 boundary components.



All triangles in $\mathrm{Sep}(S)$ are thin

Theorem (R.) When S has 2 or fewer boundary components, $\mathrm{Sep}(S)$ is relatively hyperbolic.



$\mathrm{Sep}(S)$ is hyperbolic outside of a collection of isolated regions

Geometry via Projections

$W \subseteq S$ intersects every separating curve $\implies \pi_W : \mathrm{Sep}(S) \longrightarrow C(W)$

Geometry via Projections

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$W \subseteq S$ intersects every separating curve $\implies$ $\pi_W : \mathrm{Sep}(S) \longrightarrow C(W)$

encodes interaction between separating curves and W

Geometry via Projections

Hyperbolic ↓

$W \subseteq S$ intersects every separating curve $\Longrightarrow$ $\pi_W : \mathrm{Sep}(S) \longrightarrow C(W)$

Witness

encodes interaction between separating curves and W

Geometry via Projections

$$\mathrm{Sep}(S) \longrightarrow \prod_{W \in \mathcal{W}} C(W)$$

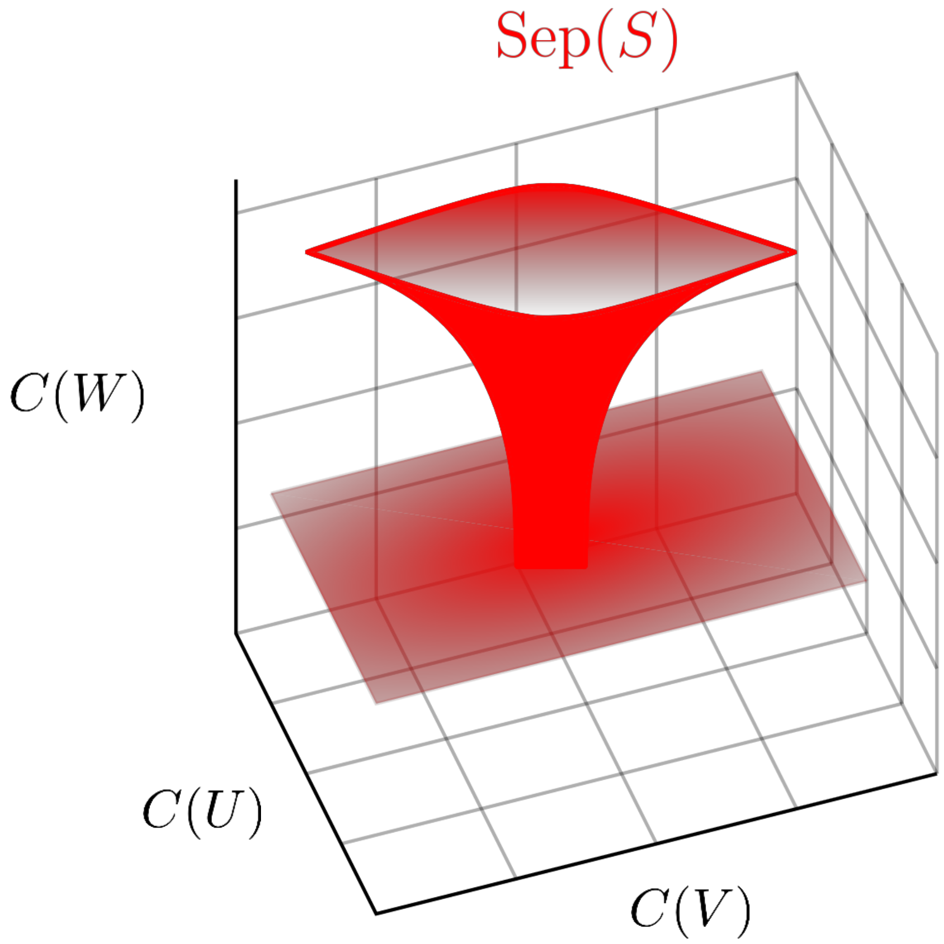
$$\gamma \longrightarrow (\pi_W(\gamma))_{W \in \mathcal{W}}$$

$\mathcal{W} = \{$subsurfaces which intersect every separating curve$\}$

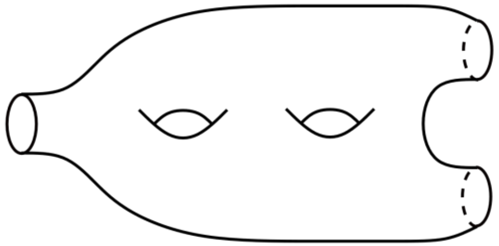
Geometry via Projections

$$\mathrm{Sep}(S) \longrightarrow \prod_{W \in \mathcal{W}} C(W)$$

Position of witnesses on S determines how $\mathrm{Sep}(S)$ sits inside $\prod C(W)$



Vokes' Argument for Hyperbolicity



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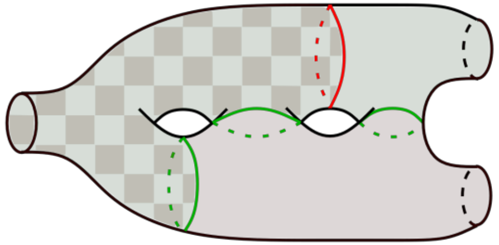

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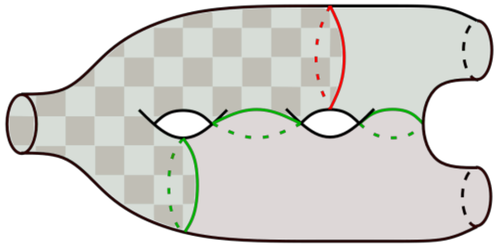
$$\Downarrow$$

$\mathcal{W}$ contains no disjoint subsurfaces

Vokes' Argument for Hyperbolicity



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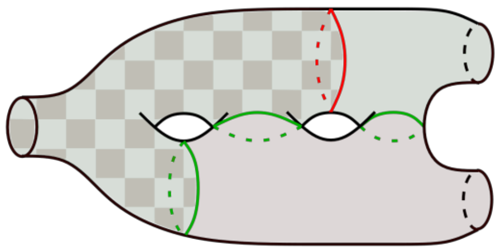
Behrstock
Hagen
Sisto

$\Downarrow$

Sep(S) hyperbolic

Vokes' Argument for Hyperbolicity



S has at least 3 boundary components

$\Downarrow$

$\mathcal{W}$ contains no disjoint subsurfaces

$\Downarrow$

Sep(S) hyperbolic

Behrstock
Hagen
Sisto

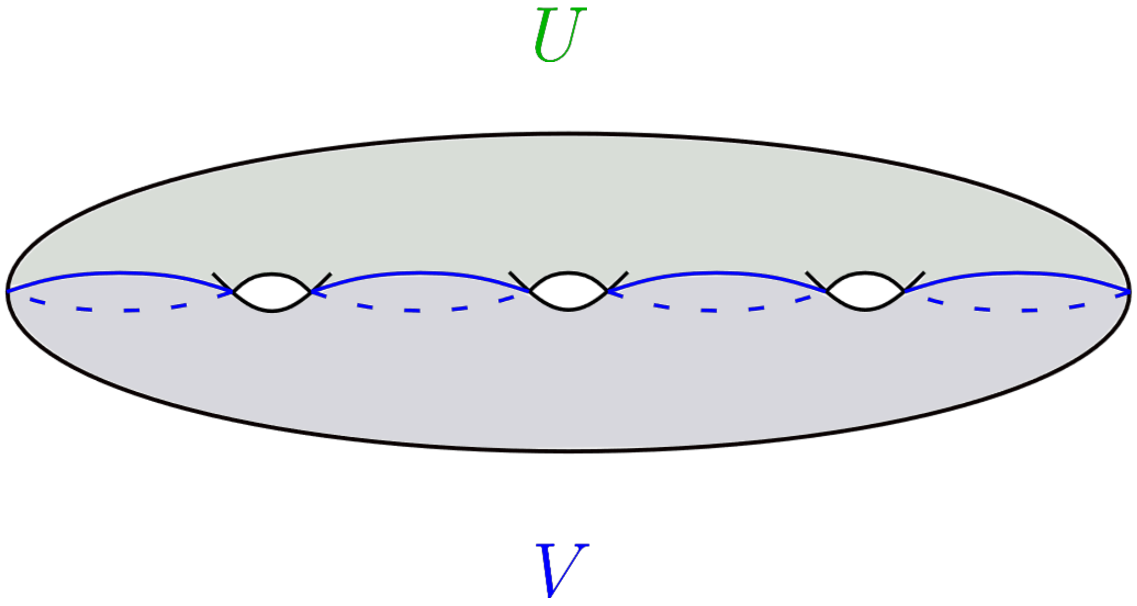
Disjoint witnesses obstruct hyperbolicity

Theorem (R.) Sep(S) is relatively hyperbolic when S is closed.

$U, V \in \mathcal{W}$ with U disjoint from V

Theorem (R.) $\mathrm{Sep}(S)$ is relatively hyperbolic when S is closed.

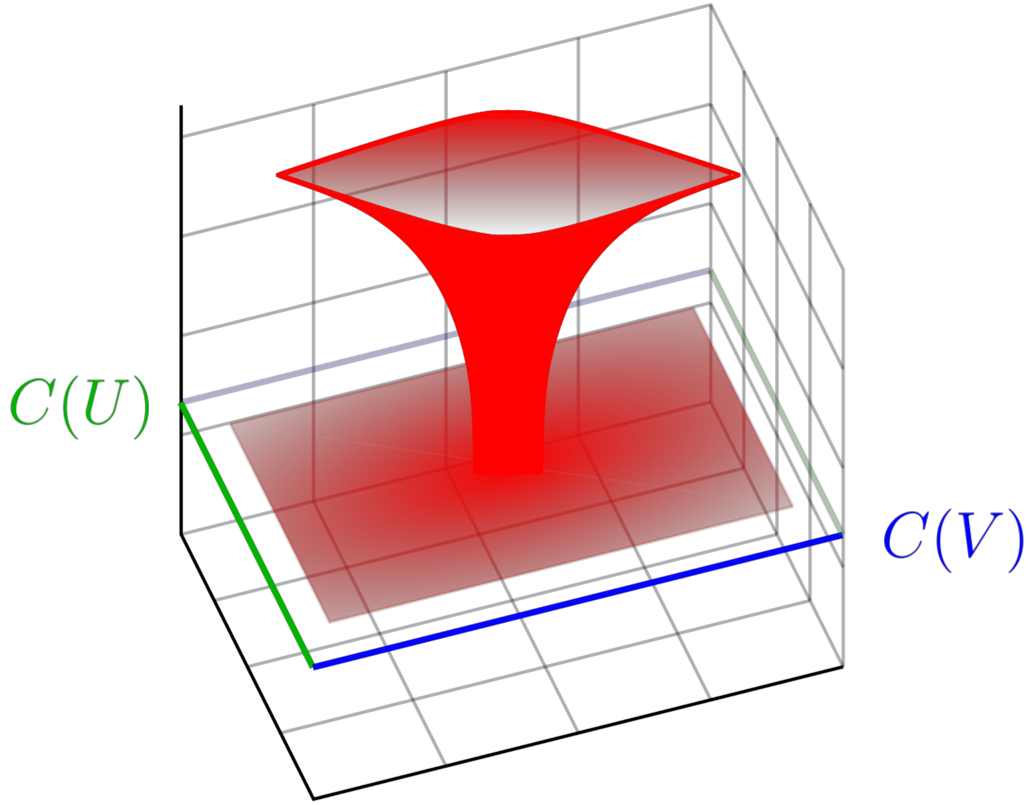
$U, V \in \mathcal{W}$ with U disjoint from V



Only configuration of disjoint witnesses for $\mathrm{Sep}(S)$

Theorem (R.) $\mathrm{Sep}(S)$ is relatively hyperbolic when S is closed.

$U, V \in \mathcal{W}$ with U disjoint from V

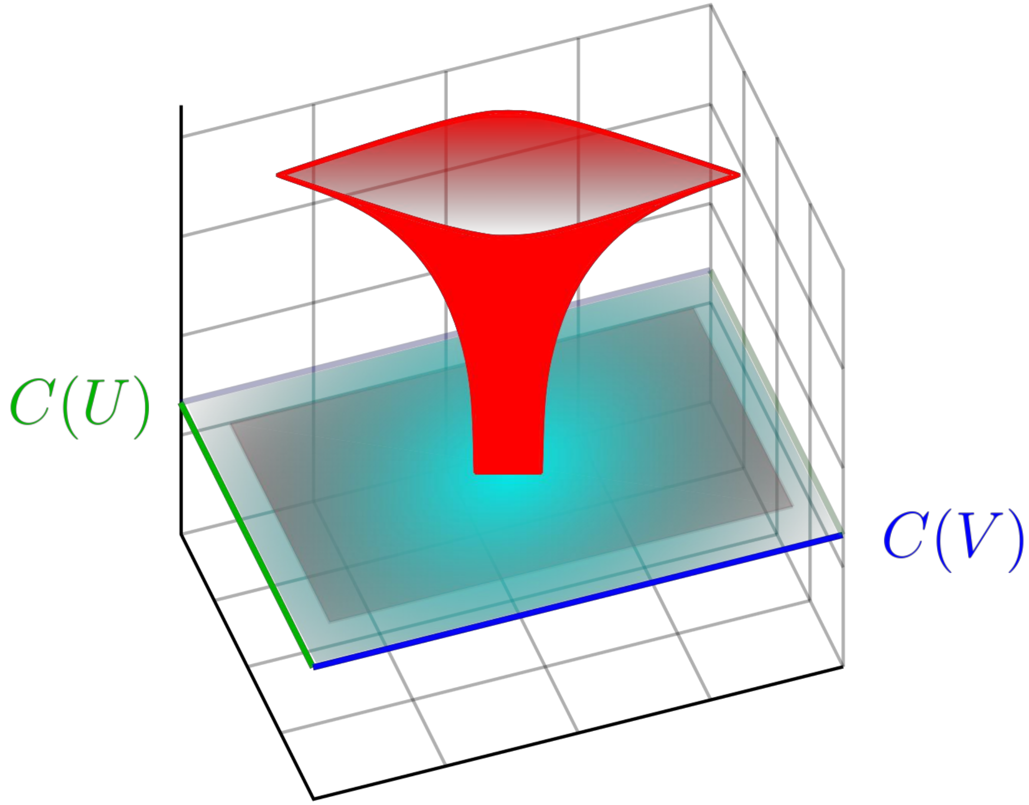


Theorem (R.) $\mathrm{Sep}(S)$ is relatively hyperbolic when S is closed.

$U, V \in \mathcal{W}$ with U disjoint from V

$$\Downarrow$$

Product region in $\mathrm{Sep}(S)$



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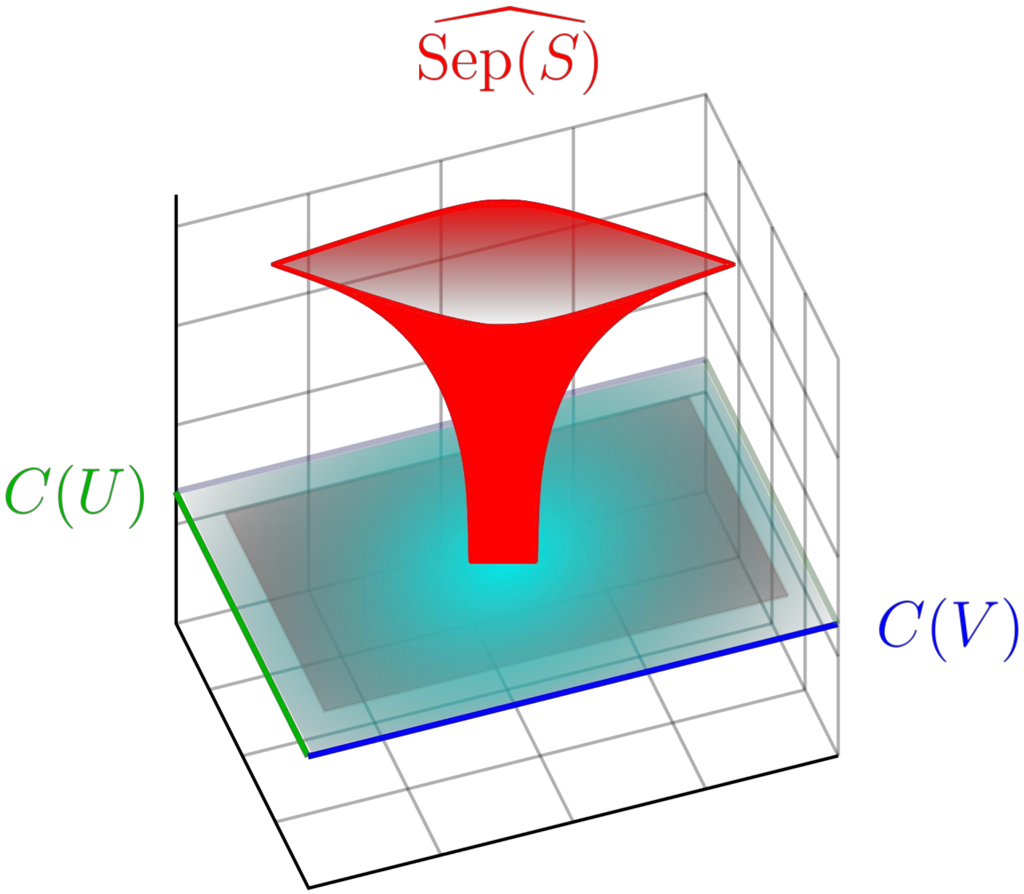


$U, V \in \mathcal{W}$ with U disjoint from V

$$\Downarrow$$

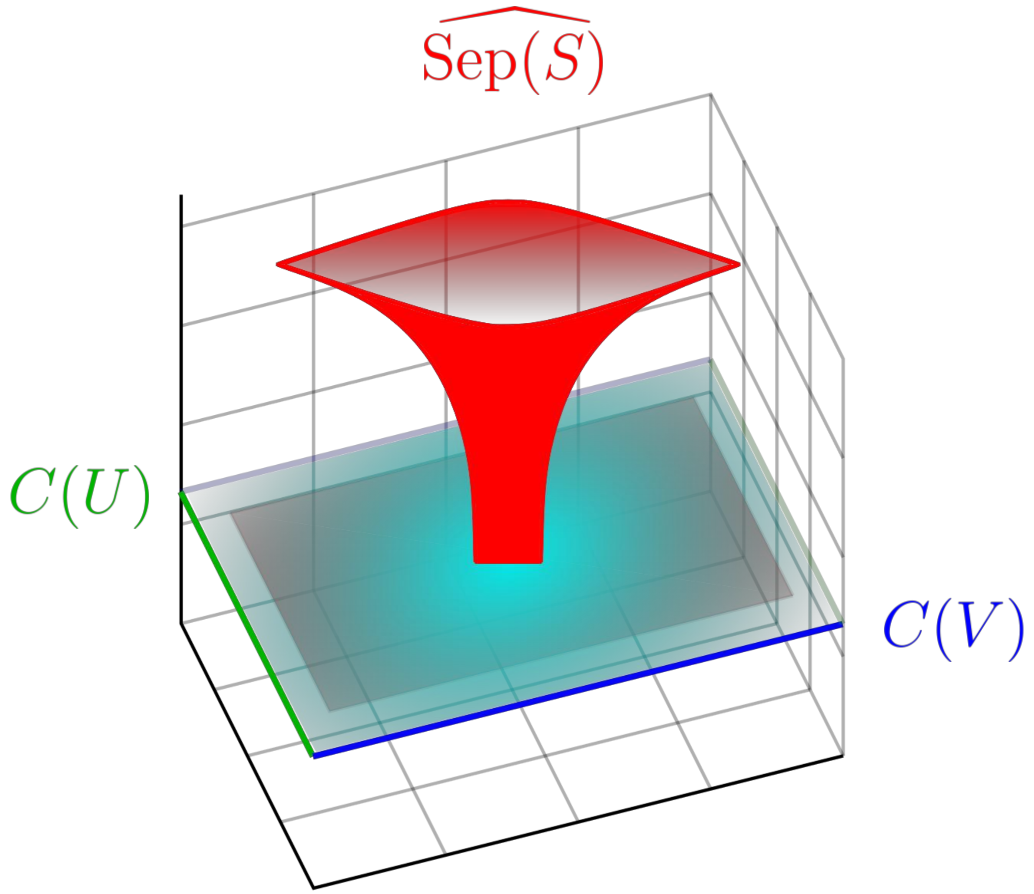
Product region in $\mathrm{Sep}(S)$

$\widehat{\mathrm{Sep}(S)} =$ Portion of $\mathrm{Sep}(S)$ outside of product regions

Theorem (R.) $\mathrm{Sep}(S)$ is relatively hyperbolic when S is closed.

$\widehat{\mathrm{Sep}(S)} =$ Portion of $\mathrm{Sep}(S)$ outside of product regions

$$\widehat{\mathrm{Sep}(S)} \longrightarrow \prod_{W \in \widehat{\mathcal{W}}} C(W)$$



Theorem (R.) Sep(S) is relatively hyperbolic when S is closed.

$\widehat{\text{Sep}(S)}$ = Portion of Sep(S) outside of product regions

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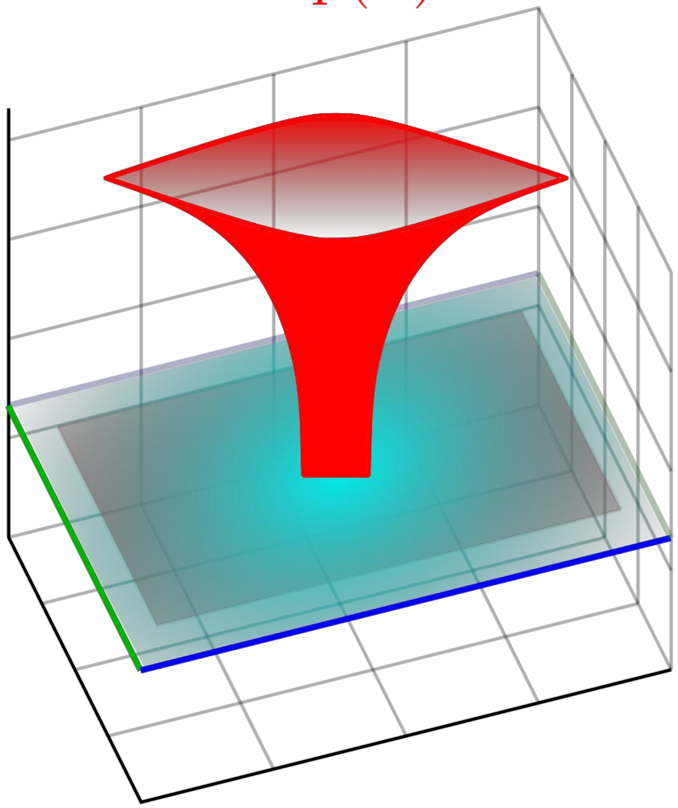
$$\widehat{\mathcal{W}} = \mathcal{W} - \{U, V : U \text{ and } V \text{ are disjoint}\}$$



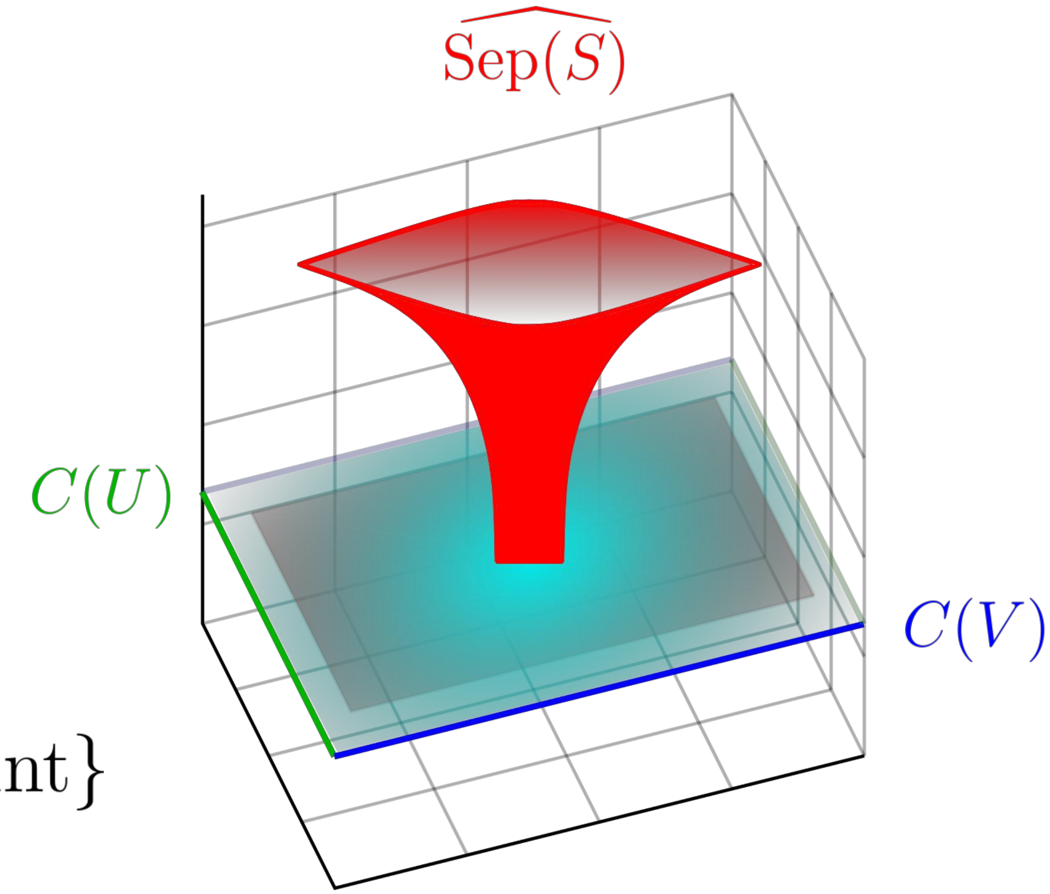
Theorem (R.) $\mathrm{Sep}(S)$ is relatively hyperbolic when S is closed.

$\widehat{\mathrm{Sep}(S)} =$ Portion of $\mathrm{Sep}(S)$ outside of product regions

$$\widehat{\mathrm{Sep}(S)} \longrightarrow \prod_{W \in \widehat{\mathcal{W}}} C(W)$$

$\widehat{\mathcal{W}} = \mathcal{W} - \{U, V : U \text{ and } V \text{ are disjoint}\}$

$\widehat{\mathcal{W}}$ contain no disjoint subsurfaces $\implies \widehat{\mathrm{Sep}(S)}$ hyperbolic



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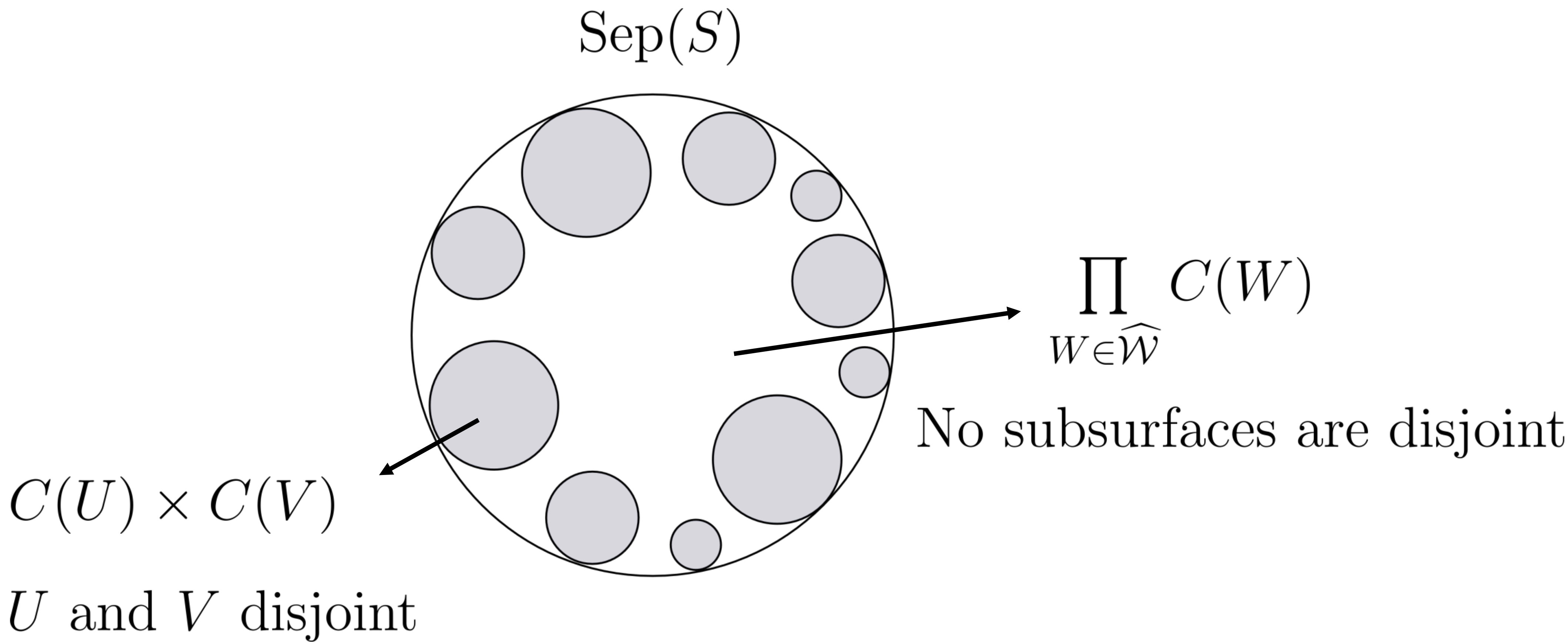

Splitting Surfaces of 2-Component Links with Multivariable Alexander Polynomial 0

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Notation and Definitions

- $L = L_1 \cup L_2 \subset S^3$ a 2-component link
- $X = S^3 \backslash \mathcal{N}(L)$
- $\rho : \widetilde{X} \to X$ be the universal abelian covering map, i.e. the one corresponding to the commutator subgroup.
- Its group of deck transformations is $H_1(X, \mathbb{Z}) \cong \mathbb{Z}^2$

The Multivariable Alexander Polynomial and $H_2(\widetilde{X}, \mathbb{Z})$

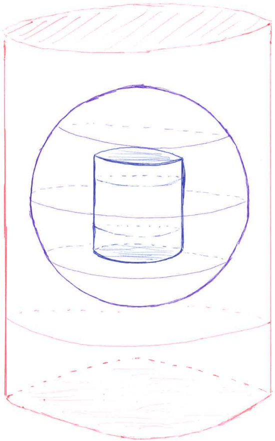
- $\Delta_{(x,y)} = 0$ if and only if $H_2(\widetilde{X}, \mathbb{Z})$ is free on one generator when regarded as a $\mathbb{Z}H_1(X, \mathbb{Z})$-module.
- We define:

  $$g_{split} = min\{genus(S) : \text{S is a surface and [S] generates } H_2(\widetilde{X}, \mathbb{Z})\}$$

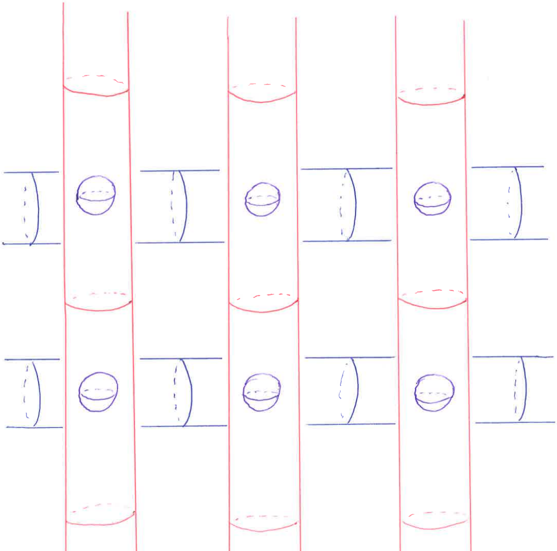
- What does g_{split} tell us about L?

Universal Abelian Cover of the 2-Component Unlink

A fundamental domain of $\widetilde{X}$ under the group action of $H_1(X, \mathbb{Z})$



Universal Abelian Cover of the 2-Component Unlink



The Genus $g_{split} = 0$ case

- Theorem: $g_{split} = 0$ if an only if L is a split link.

The Genus $g_{split} = 1$ case

- Theorem [A., Baker, in progress]: If $g_{split} = 1$, then L is a toroidal boundary link.
- The primary tools we used in this proof were the Torus theorem and the JSJ-decomposition

The Genus $g \geq 2$ case

- We can construct a surface $S \subset \widetilde{X}$ representing a generator using Fox calculus to get an upper bound for g_{split}
- Tools that were useful in the genus $g_{split} = 1$ case don't have good analogues
- In general we can expect $\rho(S)$ to be an immersed surface, but not embedded unless L is a boundary link.

Thank You!